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Condition State Based Civil Infrastructure Deterioration Model on a Structure System Level

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ABSTRACT

The successful prediction of civil infrastructure's deterioration process is crucial for making optimal maintenance, rehabilitation, and replacement (MR&R) decisions under financial constraints. The majority of current deterioration models simulate the deterioration process of a single structure element of civil infrastructure; such models thus ignore the interaction between dependent elements. However, the interaction between structure elements often plays an important role in the deterioration of the overall structure. Therefore, the primary objective of this paper is to address the interaction of these structure elements by developing a method to simulate the deterioration process of civil infrastructure on a system level. The proposed method will also provide a measure of the uncertainty of the simulation using Markov Chain Monte Carlo (MCMC) to estimate the optimal parameters of the Markov Chain and the probability distribution of those parameters. The Monte Carlo simulation is then used to generate a large number of deterioration process samples, which serve as the base of the uncertainty analysis of the simulation. The model was applied to simulate the deterioration process of a bridge element subsystem as an example application. In this example application, the model was calibrated and evaluated by the bridge inspection record collected in the Commonwealth of Virginia, USA. The results demonstrate that including the interaction between elements into the model improves the accuracy of deterioration simulation, while also reducing the uncertainty of the results. Furthermore, the proposed model is relatively easy to implement within current infrastructure management systems (IMS) compared to other methods such as neural networks and fuzzy logical models.

Author keywords: Civil Infrastructure; Deterioration Model; Markov Chain; Markov Chain Monte Carlo; Structure Element Interaction

INTRODUCTION

Civil infrastructure deterioration poses a serious challenge to public safety and the economy worldwide (Wang and Elhag, 2007; Kobayashi, Do, and Han, 2010; Sun and Gu, 2011; Setunge et. al., 2016). According to the 2017 Infrastructure Report Card provided by the American Society of Civil Engineers (ASCE), America's infrastructure is below standard and in fair to poor condition, especially as many elements approach the end of their service life (ASCE, 2017). A primary challenge in making maintenance, rehabilitation, and replacement (MR&R) decisions for civil infrastructure is due to financial constraints on infrastructure owners (Agrawal and Kawaguchi, 2009). To address this challenge, systematic and effective infrastructure management systems (IMS) are increasingly required to optimize MR&R decisions under financial constraints (Agrawal and Kawaguchi, 2009; Tran et. al., 2010). The quality of these decisions depends on successful prediction of civil infrastructure's future condition state. Prior research has developed different types of deterioration models for single structure elements such as stochastic models, neural network models, and fuzzy logical models. (Micevski et. al., 2002; Baik et. al., 2006; Kobayashi, Kaito, and Lethanh, 2010; Thomas and Sobanjo, 2016).

Stochastic models, particularly Markovian models, have been extensively used in predicting the deterioration process of civil infrastructure facilities, e.g., bridge elements (Wellalage et. al., 2015; Thomas and Sobanjo, 2016), pavements (Kobayashi, Do, and Han., 2010; Thomas and Sobanjo, 2013), and storm-water and wastewater pipes (Micevski et. al., 2002; Tran et. al., 2010). Markovian models are the most commonly used deterioration models in current IMS, for example, the AASHTOWare Bridge Management System. A primary advantage of Markovian models is that they are able to capture the physical and intrinsic uncertainty when predicting the future condition of civil infrastructures. These models are also much easier to calibrate and apply compared to other, more sophisticated, methods (Thomas and Sobanjo, 2013). However, stochastic models have several drawbacks as well, namely, they are sensitive to noisy data and they are based on assumed probability distributions (Tran et. al., 2007; Agrawal and Kawaguchi, 2009).

Free from these limitations, neural network models (NNM) have been applied to predict structure deterioration processes in many previous studies (Tran et. al., 2007; Tran et. al., 2009; Huang, 2010; Son et. al., 2010; Lee et. al. 2014). NNM is capable of analyzing problems that are poorly defined or too complex to be clearly understood (Tran et. al., 2007; Lee et. al., 2014). Meanwhile, NNM can rank input factors in order of importance to the deterioration process, which is useful for identifying the influential factors (Tran et. al., 2007).

Fuzzy logic theory, which is capable of addressing vague and uncertain problems, is another widely used method in civil infrastructure deterioration simulation (Kaufmann and Gupta, 1985; Jeong et. al., 2017). Examples using the fuzzy logic theory include pavement condition evaluation (Sun and Gu, 2011; Jeong et. al., 2017), buried pipeline deterioration simulation (Kleiner et. al., 2006; Tagherouit et. al., 2011), and bridge condition evaluation (Wang and Elhag, 2007; Tarighat and Miyamoto, 2009). However, a primary limitation with fuzzy-based models is that factors affecting the deterioration rates and inference rules are identified and constructed based on expert opinion, which can often be subjective (Tran et. al., 2007; Marzouk and Osama, 2017).

For civil infrastructure consisting of multiple elements, the interaction between elements exists because they are physically interconnected while serving different specific functions (Sianipar and Adam, 1997). Several methods have been applied to estimate infrastructure deterioration due to element interactions (Sianipar and Adam, 1997; Morcous et. al., 2002; Setunge et. al., 2016). Sianipar and Adams (1997) first used fault-tree models to simulate bridge element deterioration while considering the interactions between elements. Subsequently, fault-tree models have been successfully used by many studies to estimate the deterioration rate or failure risk of civil infrastructures (LeBeau and Wadia-Fascetti, 2007; Davis-McDaniel et. al., 2013; Setunge et. al., 2016). In these studies, the failure probability or deterioration rate of a structure is calculated based on the probabilities of a series of base events. For example, in a deteriorating bridge, the malfunction of expansions joints could be considered as a base event because the malfunction of expansions joints often accelerates the deterioration of adjacent structure elements, for example, the bearing system and bridge deck (Sianipar and Adams, 1997). When using fault-tree models, the most essential step is to estimate the probabilities of the occurrence of these base events. However, there are insufficient observed data to determine these probabilities in most cases. In addition, the base event occurrences are assumed to be independent from one another, which may not be correct in all the cases (Sianipar and Adams, 1997).

Another method that is capable of capturing the interaction between structural elements is the case-based reasoning (CBR) approach. CBR is an artificial intelligence technique that can be used to estimate the deterioration process of civil infrastructure (Morcous et. al., 2002; Waheed and Adeli, 2004). The fundamental assumption of CBR is that the deterioration process under the current situation can be treated as a similar case that happened in the historical record. When using CBR, first, a case library including the historical records of structure conditions and influence factors is built; then, the case library is searched to find the most similar stored cases to the current situation. The condition states of structure elements can be treated as the influence factors of interrelated structure elements and stored in the case library. Thus, the deterioration process of structure elements can be simulated while considering the condition states of interrelated structure elements in the case library. Limitations with the CBR include the requirement of an adequate size and coverage in the case library and the subjectivity while determining the weights of different influence factors by expert opinions.

The objective of this paper is to design a method to simulate the deterioration process of civil infrastructure on system level. In this paper, structure elements that affect the deterioration processes of other elements are named as protecting elements, and conversely, the elements being affected are base elements. A Markov Chain-based method initially proposed by Reardon (2015) was expanded to estimate structure deterioration including the interaction between structure elements. The basic assumption of Reardon’s method is that the Markov Chain transition probabilities of base elements are affected by the condition state of protecting elements. The original method works for one-to-one element dependencies. In this research, the method was expanded to capture the interaction between multiple elements. All parameters in the proposed method are calibrated from inspection records. In this paper, the deterioration process refers to network-level deterioration, i.e., the deterioration process of a large population of a specific structure element. The proposed method makes it possible for decision makers to predict the future condition state of civil infrastructure, which is important for calculating the life-cycle cost and making effective MR&R decisions. In addition, the proposed method is based on a stochastic model, which has been shown to provide better extrapolation capabilities than deterministic models that predict the future condition of bridge element based on many factors, including age, environment, design characteristics, and traffic conditions (Cavalline et. al., 2015). To predict the future conditions of civil infrastructures using the complicated methods mentioned above (i.e., NNM, fuzzy logical models, fault-tree models, and CBR), the prediction of influence factors is indispensable. However, the prediction of influence factors is usually unavailable or has large uncertainty. This makes it very hard to integrate these methods into current IMS. However, this is not a problem for stochastic models, e.g., the proposed model, because the application of these models is independent from these influence factors. This makes the proposed model easier to be integrated into current IMS.

A key limitation of prior methods is that the uncertainty of the deterioration process has not been considered. No matter what method is used, the parameters which defined the deterioration processes are inevitably affected by uncertainties associated with intrinsic randomness and imperfections of algorithms (Biondini and Frangopol, 2016). The parameters of the aforementioned methods became fixed values after model calibrated. This makes the model become stationary, i.e., a unique deterioration process would be generated given certain initial conditions, regardless of the uncertainty of the deterioration process. A solution to this problem is making full use of the probability distribution of model parameters. In this study, a Bayesian approach-based Markov Chain Monte Carlo (MCMC) model is utilized to find the optimized model parameters as well as the probability distribution of parameters. The MCMC model is widely used in calibrating model parameters and deriving the probability distribution of parameters (Micevski et. al., 2002; Hong and Prozzi, 2006; Tran et. al., 2010; Wellalage et. al., 2015). To take the uncertainties of parameters into consideration, first a large number of parameter samples are generated using MCMC and the probability distribution of each parameter is derived from these samples. Second, randomly select value of parameters according to their probability distribution, then, feed these parameters to a Monte Carlo model (Rubinstein and Kroese, 2007) to generate a large number of deterioration process instances. Finally, the uncertainty of the deterioration process is obtained by analyzing these instances.

A limitation of the current version of the proposed method exists when calculating the uncertainty of the simulation on system level. The number of the parameters of subordinate deterioration model (SDM), which is used to represent the interaction between structure elements, exceeds the limitation of MCMC when the inspection period is not long enough. Thus, the uncertainty of the interaction between structure elements are not considered in the current version of the model. In the future study, a SDM with less parameters will be developed to make sure the uncertainty can be thoroughly considered during the simulation.

The remainder of the paper is organized as follow. The methodology section provides details for implementing this method. An example application is then presented applying the method to simulate the network-level deterioration process of bridges in the Commonwealth of

Virginia. The paper concludes with a discussion of the benefits and limitations of the approach, along with possible future research to further advance the approach.

METHODOLOGY

The proposed method is capable of simulating the deterioration process of a structure element system while considering the interaction between elements. As an example, the procedure for estimating the deterioration process of a base element under the influence of a single protecting element using the proposed method is presented in Fig. 1. In this procedure, the first step is to calculate the network-level deterioration processes of the base element and protecting element based on the inspection record of each individual structure element in this network. This deterioration process is defined as observed deterioration process because it is a representation of the condition of structure element from inspection record. The deterioration processes of the base element and protecting element are then used to calibrate the subordinate deterioration model, which captures the interaction between interrelated structure elements. Based on the observed deterioration processes of the protecting element, a large number of Markov Chain parameter samples are generated using the Bayesian MCMC. The probability distributions of Markov Chain parameters are then derived from these samples. A Monte Carlo simulation is used to generate an adequate number of deterioration process instances of the protecting element. With known initial condition states, the same number of deterioration process instances of base elements are generated corresponding to the deterioration process instances of the protecting element using the calibrated subordinate deterioration model. The output is then compare with the observed deterioration process to evaluate the performance of the proposed method. Details for each step are included in the following subsections.

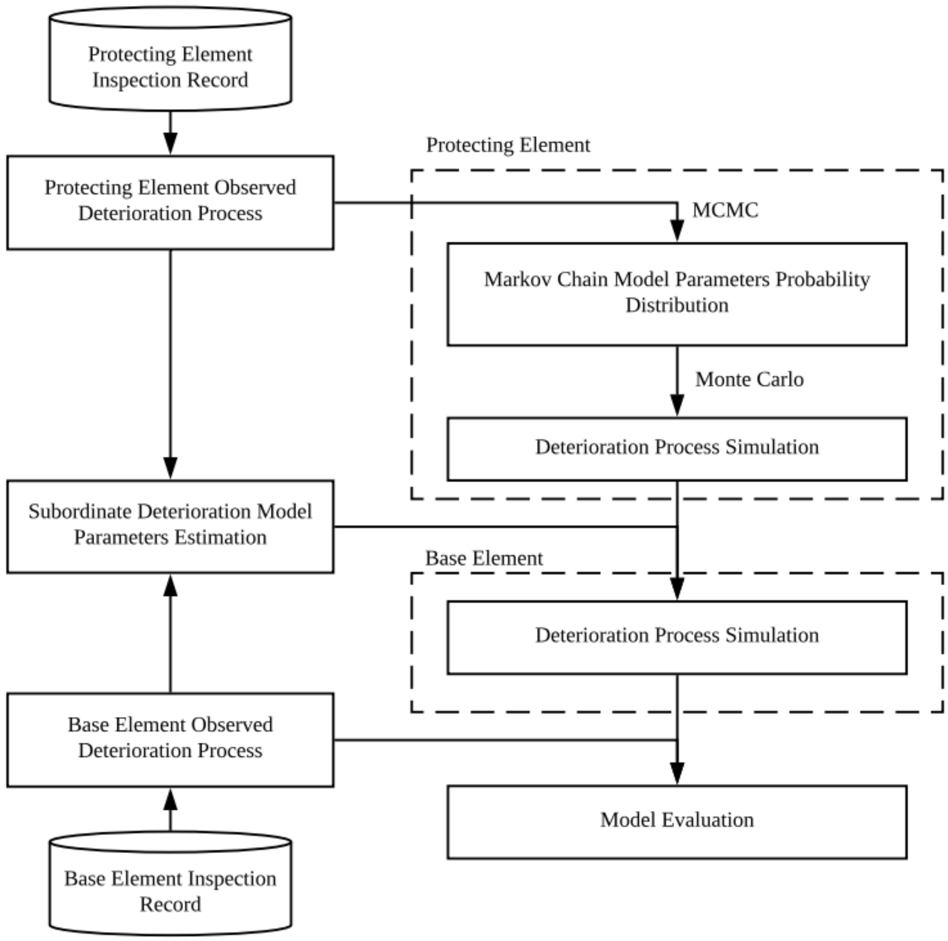

Fig. 1. Procedure to simulate base element deterioration process under the influence of single protecting element

Age-based Element Condition State Distribution

The first step is to calculate the percentage of structural elements' quantity, for example, surface area, in each condition state on a network-level from historical inspection records. Most prior approaches calculate this condition state distribution on a calendar year basis, i.e., annual time series (Tran et. al., 2010; Wellalage et. al., 2015; Thomas and Sobanjo, 2016). There are two drawbacks to using this method. First, the time series would be relatively short because the inspection record yielded by most current IMSs is less than 30 years. Second, age is an important factor on the element deterioration rate, but it is ignored in this method (Ng and Moses, 1998;

Thomas and Sobanjo, 2013 and 2016). To address these limitations, the proposed method adopts a method that the condition state distribution is calculated based on the age of structure elements when they were inspected. This age-based method for a specific structure element is given by

$$CS_i^j = \frac{\sum_{m=1}^{M}(q_i^j)_m}{\sum_{i=1}^{N}\left[\sum_{m=1}^{M}(q_i^j)_m\right]} \times 100\% \tag{1}$$

where, CS_i^j is the percentage of the overall quantity in condition state i at the age of j, M is the total number of this type of structure element inspected at the age of j, $(q_i^j)_m$ is the quantity of element m in condition state i at the age of j, and N is the total number of condition states.

Markov Chain

Markov Chain is widely used in current civil infrastructure management systems. A simplified Markov Chain transition probability matrix for stationary structure element deterioration is shown in Equation (2). Compared to an ordinary Markov Chain, Equation (2) is simplified in following two points. First, all values below the main diagonal are zero because the structure condition cannot be improved without MR&R actions. Second, the probability of an element decaying by more than one condition state is zero between two successive inspections. McCalmont (1990) showed that the probability of having more than one condition state jump is negligible. The transition probability matrix is given by

$$\mathbf{TPM} = \begin{bmatrix} P_{1,1} & 1-P_{1,1} & 0 & \cdots & 0 & 0 \\ 0 & P_{2,2} & 1-P_{2,2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & P_{N-1,N-1} & 1-P_{N-1,N-1} \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{2}$$

where **TPM** is the transition probability matrix, $P_{i,i}$ is the probability of an element staying in condition state i between two successive inspections, and N is the total number of condition. For a given initial condition state, $\boldsymbol{CS}_0$, and $\mathbf{TPM}$, the condition state distribution at age *n* can be found using Equation (3):

$$\mathbf{CS}_n = \mathbf{CS}_0 \times \mathbf{TPM}^n \tag{3}$$

Bayesian Markov Chain Monte Carlo Simulation

Bayesian Approach

From Bayesian theory, the calibration of an unknown parameter vector $\boldsymbol{\theta}$ is an update from its prior distribution using known information through some probabilistic model (Yuan et. al., 2009). In this paper, the known information is the observed condition state distribution, $\mathbf{CS} = \{cs_1, cs_2, \ldots, cs_n\}$, and the unknown parameter vector $\boldsymbol{\theta}$ equals the main diagonal of Equation 2, i.e.,

$$\boldsymbol{\theta} = \left[P_{1,1}, P_{2,2}, \cdots, P_{N-1,N-1}, 1\right] \tag{4}$$

According to Bayes' theorem, the posterior distribution of model unknown parameters is given by

$$P(\boldsymbol{\theta}|\mathbf{CS}) = \frac{L(\mathbf{CS}|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\int P(\mathbf{CS}|\boldsymbol{\theta})P(\boldsymbol{\theta})d\boldsymbol{\theta}} = \frac{L(\mathbf{CS}|\boldsymbol{\theta})P(\boldsymbol{\theta})}{P(\mathbf{CS})} \tag{5}$$

where $P(\boldsymbol{\theta}|\mathbf{CS})$ is the posterior distribution of $\boldsymbol{\theta}$ given observed data $\mathbf{CS}$, $L(\mathbf{CS}|\boldsymbol{\theta})$ is the likelihood to observed $\mathbf{CS}$ given unknown parameters $\boldsymbol{\theta}$, $P(\boldsymbol{\theta})$ is a prior probability distribution representing the initial beliefs about the true value of $\boldsymbol{\theta}$, and $P(\mathbf{CS})$ is the probability distribution of $\mathbf{CS}$. Because $P(\mathbf{CS})$ is independent of $\boldsymbol{\theta}$, the posterior distribution is proportional to the product of prior distribution density and the likelihood function as given by

$$P(\boldsymbol{\theta}|\mathbf{CS}) \propto P(\boldsymbol{\theta})L(\mathbf{CS}|\boldsymbol{\theta}) \tag{6}$$

Because there is no available knowledge about the prior distribution of these Markov Chain parameters, the prior distribution $P(\boldsymbol{\theta})$ was chosen as a uniform distribution in interval [0, 1]. As a result, the posterior distribution $P(\boldsymbol{\theta}|\mathbf{CS})$ is proportional to the likelihood function $L(\mathbf{CS}|\boldsymbol{\theta})$.

With a randomly selected $\boldsymbol{\theta}$ in the space [0, 1] and a known initial condition state distribution, the deterioration process can be calculated using a Markov Chain simulation. Then, for each specific element age, the error between the simulation and observation can be computed by using a Half-Normal Distribution method (Bland, 2005), which treats the difference between the simulation and observation as a probability. The probability that the estimated condition state i at year t, $(CS')_i^t$, is equal the observation, CS_i^t, is expressed by the probability density function (PDF) of a Half-Normal Distribution, as follows

$$P(\boldsymbol{\theta})_i^t = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} exp\left(-\frac{[(CS')_i^t - CS_i^t]^2}{2\sigma^2}\right) \quad (7)$$

where $P(\boldsymbol{\theta})_i^t$ is the probability that the estimation of condition state i at age t is accurate by using a randomly selected parameter vector $\boldsymbol{\theta}$, and σ is a scale parameter. The value of σ would not significantly affect the result of the MCMC simulation, but it influences the stability of the simulation. Thus, a sensitivity test needs to be done to choose an appropriate σ. A sensitivity test for the example application in this paper indicates that the model for this specific case is stable while choosing the σ value in the interval [0.1, 0.3]. According to joint probability theory, the likelihood function can be calculated by

$$L(\mathbf{CS}|\boldsymbol{\theta}) = \prod_{t=1}^{T}\prod_{i=1}^{N} P(\boldsymbol{\theta})_i^t \quad (8)$$

where T is the maximum element age in the study period and N is the number of condition states in the inspection system.

Markov Chain Monte Carlo Simulation

The Metropolis-Hastings (MH) algorithm is used to generate samples of Markov Chain parameters. The MH algorithm is one of the most established and commonly used MCMC algorithms (Green and Worden, 2015). Throughout the following text, a target distribution is defined by Equation (9).

$$\pi(\boldsymbol{\theta}) = P(\boldsymbol{\theta})L(\mathbf{CS}|\boldsymbol{\theta}) \tag{9}$$

At each iteration, a candidate sample $\boldsymbol{\theta}'$ is randomly selected from a uniform distribution in space [0, 1]. Then, the deterioration process is simulated with a known initial condition state. Given a condition state observation, $\mathbf{CS}$, the target distribution $\pi(\boldsymbol{\theta}')$ can be calculated. This target distribution $\pi(\boldsymbol{\theta}')$ is then subject to an acceptance test with target distribution $\pi(\boldsymbol{\theta}_i)$ for current Markov Chain parameters vector $\boldsymbol{\theta}_i$. This acceptance test is based on Equation (10).

$$\rho = \min\left\{1, \frac{\pi(\boldsymbol{\theta}')}{\pi(\boldsymbol{\theta}_i)}\right\} \tag{10}$$

If $\rho = 1$, the candidate sample $\boldsymbol{\theta}'$ is accepted and set $\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}'$; otherwise, set $\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i$. The initial starting value for the MH algorithm was randomly selected from a uniform distribution in space [0, 1]. After the MH algorithm iterates a large number of times and a certain number of "warm up" iterations at the beginning are ignored, the outputs can be used to derive the probability distribution of Markov Chain parameters.

Monte Carlo Simulation

To capture the uncertainty of the deterioration process, the Monte Carlo simulation is used to generate a large number of deterioration process instances based on the estimated probability distribution of Markov Chain parameters.

The Monte Carlo simulation in this paper consists of three basic steps.

Step1. Randomly select a value for each $P_{i,i}$ in Equation 2 according to its estimated probability distribution, then generate the **TPM**.

Step 2. Calculate the Markov Chain deterioration process start from the known initial condition state according to bridge element inspection.

Step 3. Store the simulated deterioration process, then repeat steps 1-2 a large number of times.

Element Deterioration on a System Level

To consider the interaction between structure elements, a method developed by Reardon (2015) is used in this paper. Their method is capable of capturing the interrelationship between two elements and is extended in this research to calculate the deterioration process of a base element under the influence of multiple protecting elements. This method can be applied to simulate the deterioration process of a structure element system.

Subordinate Deterioration Model

The subordinate deterioration model, developed by Reardon (2015), is used to calculate the **TPM** of the base element under the influence of a protecting element. This is a Markov Chain-based model based on the simplified form of **TPM** in Equation 2. This model assumes that the transition probability of the base element has a linear relationship with the percentage of the protecting element's quantity, for example, surface area, in each condition state. A parameter matrix is introduced into this model to compute the **TPM** of the base element. The main diagonal of the base element's **TPM** is calculated by

$$\mathbf{CTP} = \begin{bmatrix} CP_{1,1} \\ CP_{2,2} \\ \vdots \\ CP_{n-1,n-1} \\ 1 \end{bmatrix} = \begin{bmatrix} cs_1^* & cs_2^* & \cdots & cs_n^* \end{bmatrix} \begin{bmatrix} PM_{1,1} & PM_{1,2} & \cdots & PM_{1,n-1} & 1 \\ PM_{2,1} & PM_{2,2} & \cdots & PM_{2,n-1} & 1 \\ \vdots & \vdots & \ddots & \vdots & 1 \\ PM_{m,1} & PM_{m,2} & \cdots & PM_{m,n-1} & 1 \end{bmatrix} \quad (11)$$

$$E.g.: CP_{i,i} = cs_1^* \cdot PM_{1,i} + cs_2^* \cdot PM_{2,i} + \cdots + cs_m^* \cdot PM_{m,i}$$

where **CTP** equals to the main diagonal of the conditional transition probability matrix of the base element, $CP_{i,i}$ is the conditional probabilities of staying in condition state i during one time step, cs_i^* is the percentage of protecting element in condition state i, $PM_{i,j}$ is a component of the parameter matrix relates the **CTP** of base element and the condition state of protecting element, and "m" is the total number of condition states.

The parameter matrix is driven from inspection records of the base element and protecting element. This is done as follows. First, each unknown in the parameter matrix is assigned a random value in the space [0, 1]. Second, the corresponding **CTP** is calculated using Equation 11. Third, the deterioration process of the base element is calculated using Markov Chain. Finally, the Solver tool in the Microsoft Excel is used to find the optimized parameters matrix that minimizes the root-mean-square error (RMSE) between the estimated deterioration process and the observation.

Deterioration on System Level

The method to simulate the deterioration process of a civil infrastructure element under the influence of multiple elements is explained as follows. Start from a simple case that one base element is affected by *M* protecting elements. Define an array of parameters $[\lambda_1, \lambda_2, \cdots, \lambda_M]$ as the influence weight of each of these protecting elements, respectively. The conditional transition probability of the base element is given by

$$\mathbf{CTP} = \lambda_1 \cdot \mathbf{CS}_1^* \mathbf{PM}_1 + \cdots + \lambda_M \cdot \mathbf{CS}_M^* \mathbf{PM}_M \quad (12)$$

where **CTP** is the main diagonal of the conditional transition probability matrix of the base element, $\mathbf{CS}_i^*$ is the condition state distribution of protecting element i, and $\mathbf{PM}_i$ is the parameter matrix corresponding to protecting element i. The procedure for calculating the conditional **TPM** of the base element is done by the following steps.

Step 1. Separately compute the optimized parameter matrix, **PM**, corresponding to each pair of protecting element and base element using the method in the previous subsection.

Step 2. Assign each λ a random value in [0, 1], and calculate the corresponding **CTP** and **TPM** of the base element.

Step 3. Calculate the deterioration process of the base element using Equation 3.

Step 4. Find the optimized combination of $[\lambda_1, \lambda_2, \cdots, \lambda_M]$ that minimizes the RMSE between the estimated and observed deterioration process.

The method is able to be applied to calculate the deterioration process of a structure element system. Basically, the deterioration process of the system is calculated from bottom to top, i.e., the deterioration process of protecting elements would be computed at first followed by the base elements. Then, the calculated base elements become the protecting elements to simulate the deterioration processes of base elements on upper layer. This procedure will be further explained in the Example Application section. In this method, the feedback from base element is ignored. For instance, joints on a bridge structure affect the deterioration process of moveable bearings, and this relationship can be captured by the proposed method. But the feedback from moveable bearings affecting joints on bridge structures would not be counted by this method in its current form.

EXAMPLE APPLICATION

Bridges are vital components of surface transportation infrastructure. Bridges consist of many structure elements that are physically interconnected but have different specific functions (Sianipar and Adams, 1997). The interaction between bridge elements is important when modeling the deterioration processes. This interaction between bridge elements can be captured by the proposed method. To demonstrate this point, the method was applied to a set of interdependent bridge elements using data from the bridge inspection database provided by the Virginia Department of Transportation (VDOT).

Data Source

The VDOT bridge inspection database contains bridge element inspection records of 22,922 bridges and large culverts in Virginia from 1995 to 2016. According to this database, 110 bridge elements are inspected about every 2 years. The majority of Virginia's bridges were designed with an anticipated service life of 50 years, and about 64.0% of the inventory is more than 40 years old (VDOT, 2016 and 2017). Currently, the Pontis Bridge Management System (BMS) is used to manage VDOT's bridge inspection records. The Pontis BMS is a database system containing bridge element inspection records, traffic needs, accident data, maintenance records, improvement and replacement costs, etc. (VDOT, 2007). In the Pontis BMS, each bridge element is rated according to its condition state. There are two different rating systems: one that rates condtion using number 1 to 3, where 1 is the best condition and 3 is the worst condition, another that rates condition using number 1 to 5, where 1 is the best condition and 5 is the worst condition. (VDOT, 2007). Unlike the National Bridge Inventory (NBI), which assigns an overall rating to indicate the general condition of the element, the Pontis BMS rates each bridge element according to its various portions, such that, if a bridge element has multiple portions that are in different condtion states, each portion of the element will be assigned the appropriate condtion rating. For example, if 80% of the total surface area of a concrete deck is in condition state 1 and 20% is in condition state 2, the ratio 0.8 and 0.2 are assigned to condition states 1 and 2, respectively.

Study Case Description

The proposed method is applied to a subset of a bridge element system (Fig. 2) and the results are compared with results from approaches that do not consider element interactions. This system consists of five major structural elements from the bridge superstructure and bearing systems. Basic information about these elements are provided in Table 1. Detailed information about these elements can be found in the Element Data Collection Manual (VDOT, 2007). Fig. 2 shows the interdependencies between the bridge elements being studied. The relationship between each pair of interdependent elements is represented by an arrow, where the tail of an arrow is linked to a protecting element and the head of the arrow is pointing to the base element. For example, the arrow connecting Element 301 and Element 107 indicates that Element 301 is the protecting element of Element 107. In this system, the deterioration rate of movable bearings (Element 311) are affected by the conditon state of the joint seal (Element 301). This is because movable bearings are usually installed below joints and their deterioration rate can be accelarated by leakage of salt and polluted water caused by malfunctioning joints. The deterioration rate of steel open girders are also influenced by the joint seal because leaking deck expansion joints allow salt water seepage and, subsequently, corrode the girder ends. Also, the mulfunction of fixed or movable bearings by corrosion resists horizontal or vertical movement and thus accelerates the deterioration of steel open girders. As one of the main components of the deck-supporting system, girders have significant influence on the deterioration of deck system. Therefore, the girder-deck relationship is analyzed in this study.

In the inspection database, there are 476 bridges that contain the 5 elements being studied and a total of 3270 inspection records for each element in the network from 1995 to 2016. The proposed model is calibrated and tested by using the bridge element inspection from all the 476 bridges. The bridge population was randomly separated into two subsets: a training bridge set and a testing bridge set. The training bridge set contains 333 bridges (70%), and the testing bridge set includes 143 bridges (30%). All parameters in the proposed model are calibrated from the inspection records of the training bridge set. The inspection records of the testing bridge set are then used to evaluate the performance of the proposed model.

Table 1. Bridge Elements Studied and the Number of Condition States

Element	Description	No. of Condition States
12	Concrete Deck - Bare - with Uncoated Reinforcement	5
107	Steel Open Girder - Coated	5
301	Pourable Joint Seal	3
311	Moveable Bearing	3
313	Fixed Bearing	3

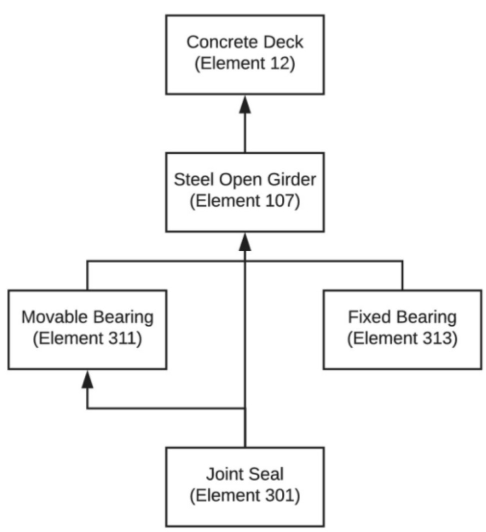

Fig. 2. Bridge element system consists of major structural components of bridge superstructure and bearing system

In this study, the condition state distributions of bridge elements are calculated based on their age when they were inspected. The proposed method is developed to simulate the deterioration process of infrastructure on a network-level. When there are too few bridges inspected at a specific age, the calculated network-level condition state distribution cannot represent the overall condition state of the bridge network at that age. Take the condition state of Element 107 on the training bridge set as an example (Fig. 3). In Fig. 3(a), a small number of bridges were inspected when they were younger than 16 years old or older than 46 years old. This results in the unstable condition state distribution when Elements 107 were at that period, which can be found in Fig. 3(b). The Element 107 between ages 16 to 46 has a relatively large bridge population inspected. At the same time, a stable deterioration process was observed. The deterioration processes of other elements are provided as the Supplemental Data to this paper. Similar to Element 107, the deterioration process of Element 301 is stable between age 16 to 46 (Fig. S1 in the Supplemental Data). In Figs. S2 and S3, the Element 311 and 313 have stable deterioration processes from age 16 to age 49. In Fig. S4, the deterioration process of Element 12 is stable from age 16 to age 48. Thus, to ensure the deterioration processes are stable for all elements considered in this study, the deterioration processes from age 16 to 47 are selected for the example application.

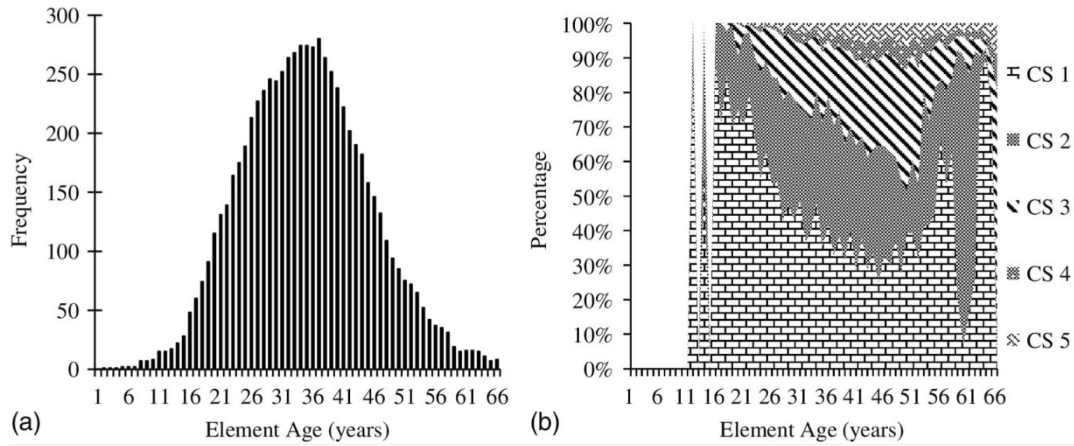

Fig. 3. Element 107 in the training bridge set (a) frequency analysis of bridge number on each age and (b) condition state distribution

Results and Discussion

Markov Chain Parameters Calibrated Using Bayesian MCMC

Starting with a set of initial Markov Chain parameters randomly selected from a uniform distribution in space [0,1], the MCMC simulation with MH algorithm was performed with 80,000 iterations for each bridge element. Trace plots with 50,000 iterations after 30,000 warm-up runs are provided for each bridge element. In this section, the MCMC simulations of Element 107 and 12 are provided as an example.

Element 107

The calibration of Markov Chain parameters of Element 107 is shown in Fig. 4(a). It can be found that the mean of the $P_{1,1}$, $P_{2,2}$, and $P_{3,3}$ simulation converges at a constant value. The simulation of $P_{1,1}$, $P_{2,2}$, and $P_{3,3}$ can be used to derive the probability distributions of $P_{1,1}$, $P_{2,2}$, and $P_{3,3}$ (Fig. 4(b)). However, the simulation of $P_{4,4}$, which affects the calculation of the CS_4 and CS_5, does not converge at any constant value. This makes it impossible to compute the optimized value and possibility distribution of $P_{4,4}$ using the Bayesian MCMC method. During the simulation period, a small percentage (about 2.5% on average) of Element 107 is observed on CS_4 and the same percentage is on CS_5. Meanwhile, the initial value of CS_4 and CS_5 usually equal zero when the element is on a good condition at the beginning of the simulation period. Thus, the simulation of CS_4 and CS_5 would be very close to zero regardless of the value of $P_{4,4}$ if the simulation period is relative short. This means the simulation of CS_4 and CS_5 is insensitive to the value of $P_{4,4}$. Therefore, under this situation, $P_{4,4}$ cannot be calibrated by using Bayesian MCMC when a very small percentage of an element's quantity are on CS_4 and CS_5. In the simulation, a default value was assigned to $P_{4,4}$ since the result would not be significantly affected by the value of $P_{4,4}$.

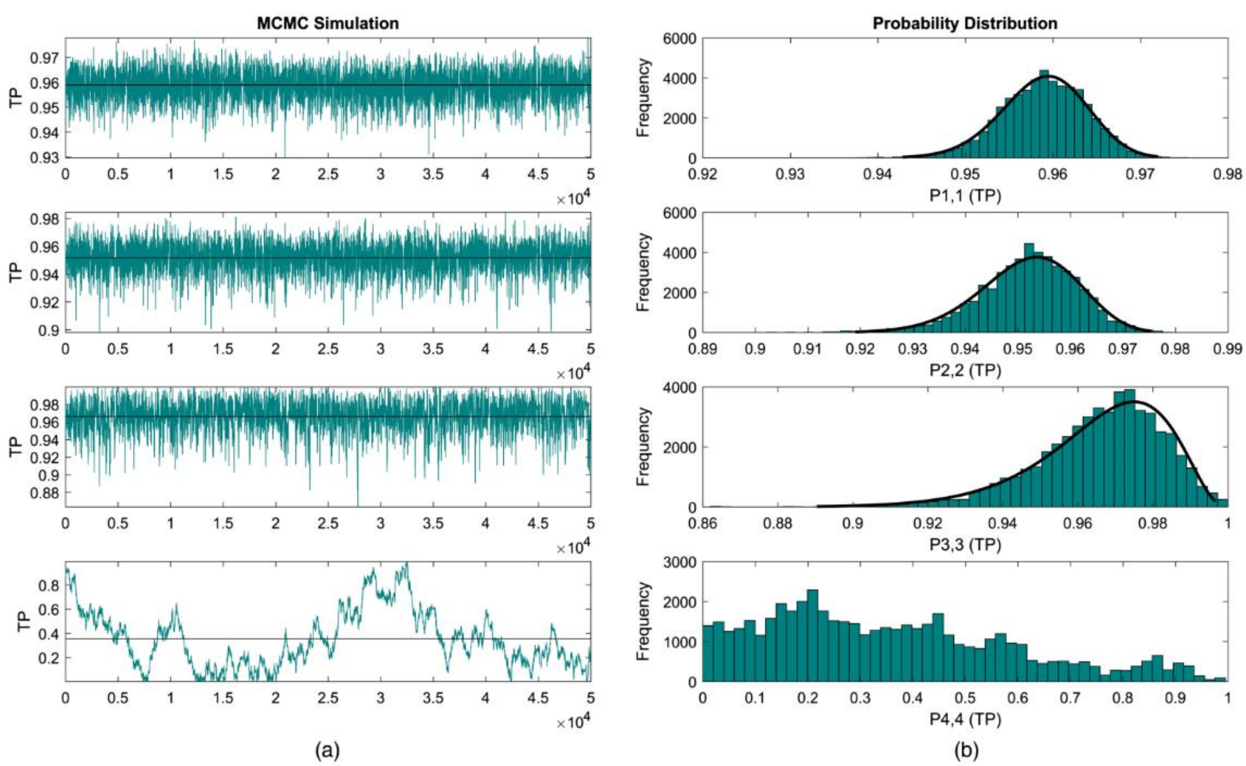

Fig. 4. (a) Markov Chain Monte Carlo (MCMC) simulation trace plot and (b) parameter probability distribution analyses of element 107

Fig. 4(b) shows the probability distribution analyses of Markov Chain transition probabilities. It can be seen that the simulations have distributions with nonzero skewness, especially $P_{3,3}$. Also, because the transition probabilities are defined in an interval of finite length ([0, 1]), the posterior distribution of TPs are assumed to be a Beta Distribution (Gupta and Nadarajap, 2004) given by

$$f(x;\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \qquad (2)$$

where α and β are two positive shape parameters and B is a normalization constant determined by α and β to ensure that the total probability integrates to 1. The Kolmogorov-Smirnov test (K-S test) (Kanji, 2006) is performed to validate the assumption that TPs follows a beta distribution. The results of the K-S test are provided in Table 2. The *h* value is the hypothesis test result, returned as a logical value. When *h* equals 1, the K-S test rejects the null hypothesis at the 0.05 significance level. Otherwise, the K-S test fails to reject the null hypothesis at the 0.05 significance level. The *p* value is the probability of observing a test statistic as extreme as the observed value under the null hypothesis. The *cv* value is the critical value at the 0.05 significance level. If *p* is smaller than *cv*, *h* would equal 1 and vice versa. In Table 2, all *h* values are equal to 1, which means that all TPs pass the K-S test and follow a beta distribution. The value of shape parameters α and β for each TP are included in Table 2. The “Mean” column is the average of TPs’ simulation in Fig. 4. The “Optimal” column contains the optimal TPs, which minimize the *RMSE* of the condition state simulation. The optimal TPs are computed by using the Solver tool in Microsoft Excel. It can be seen that there is a very small difference between the mean of TP simulations and the optimal values. The optimal value of $P_{4,4}$ in the “Optimal column” is assigned as the default value of $P_{4,4}$, which will be used to simulate the deterioration process of Element 107 along with the calibrated $P_{1,1}$, $P_{2,2}$, and $P_{3,3}$.

Table 2. Kolmogorov-Smirnov (K-S) Test and Probability Distribution of Element 107's Transition Probabilities

Transition Probabilities	Mean	K-S Test			Beta Distribution Parameters		Optimal
		h	***p***	***cv***	***α***	***β***	
P11	0.9589	1	~0	0.0061	1597.2	68.5	0.9583
P22	0.9519	1	~0	0.0061	492.1	24.9	0.9518
P33	0.9659	1	~0	0.0061	101.5	3.6	0.9660
P44	*0.8892						0.8892

Note: * is the default value of transition probability

Element 12

The trace plots and probability distribution analyses of Element 12's transition probabilities are shown in Fig. 5. The mean of $P_{1,1}$, $P_{2,2}$, and $P_{3,3}$ converge at a constant value, but the mean of $P_{4,4}$ does not converge. The K-S test is applied to verify the assumption that the transition probabilities follows a beta distribution. The results are provided in Table 3. All *h* values are equal to 1, which means $P_{1,1}$, $P_{2,2}$, and $P_{3,3}$ pass the K-S test at the 0.05 significance level. The shape parameters of beta distribution are included in Table 3. Table 3 also contains the mean of the TP simulated by Bayesian MCMC and the optimal TP values computed by using Excel Solver. Similar to Element 107, the $P_{4,4}$ value in the "Optimal" column is assigned as the default value of $P_{4,4}$.

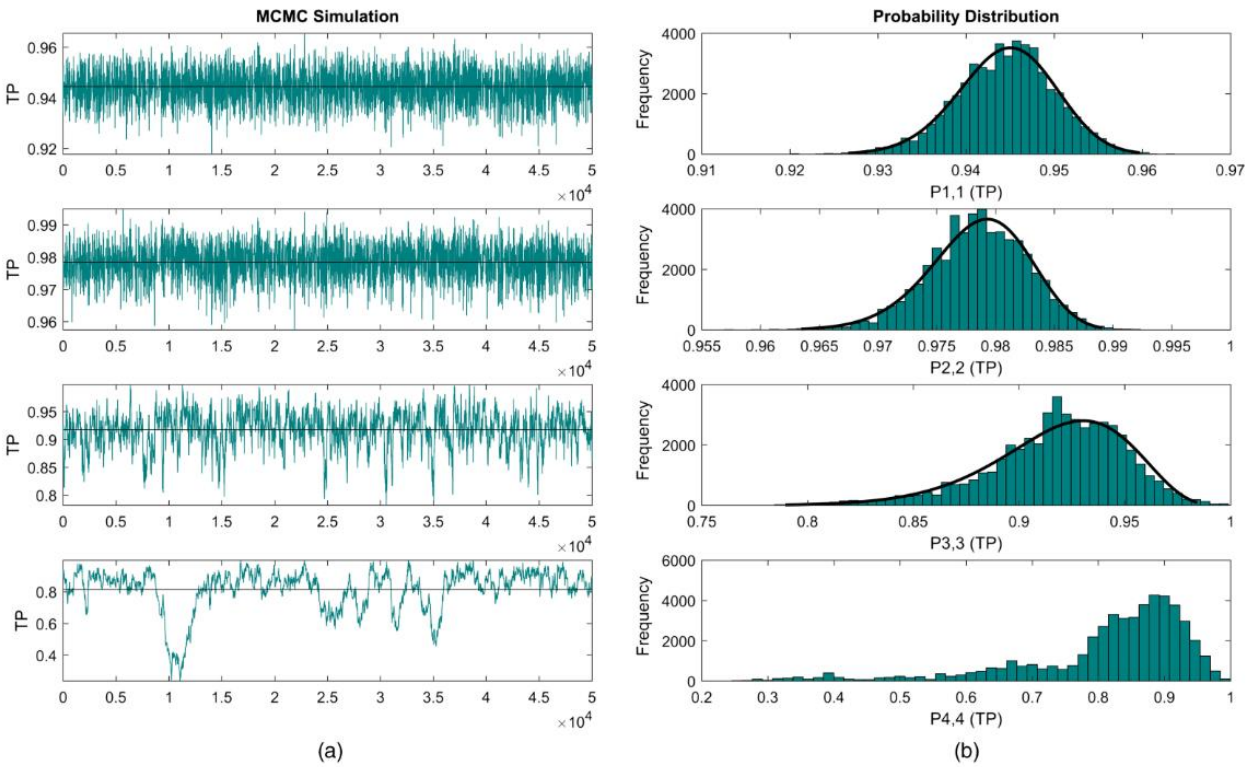

Fig. 5. (a) MCMC simulation trace plot and (b) parameter probability distribution analyses of Element 12

Table 3. K-S Test and Probability Distribution of Element 12's Transition Probabilities

Transition Probabilities	Mean	K-S Test			Beta Distribution Parameters		Optimal
		h	***p***	***cv***	***α***	***β***	
P11	0.9445	1	~0	0.0061	1625.5	95.5	0.9428
P22	0.9785	1	~0	0.0061	1130.7	24.9	0.9774
P33	0.9178	1	~0	0.0061	63.1	5.7	0.9208
P44	*0.8816						0.8816

Note: * is the default value of transition probability

Deterioration Process Simulation on a Single Element Level

Starting with the known initial condition state, the deterioration process of a single bridge element was simulated by using the Monte Carlo model. The Monte Carlo model iterated 5,000 times for each bridge element. On each iteration, the transition probabilities are randomly selected from the beta distributions derived in the previous subsections. As an example, the results of Element 107 and 12 are provided and discussed below.

Element 107

The Monte Carlo simulation of Element 107 is presented in Fig. 6. The solid lines represent the mean of the simulation at each age, and the dashed lines are the observed deterioration processes. It can be seen that the mean of the simulations are consistent with the observed deterioration processes, especially in the period when the element is older than age 20. The gray bands represent the space between the maximum and minmum percentage of bridge element quantity at each age. As a whole, the observed deterioration process is covered by the gray bands except condition state 1 and 2 at the beginning of the study period. These spaces represent the uncertainty of the deterioration process simulation, which is important information for decision makers. The width of these bands grows with the increase of the bridge element age. This means that the uncertainty of the model is growing with the increase of the length of the simulation period.

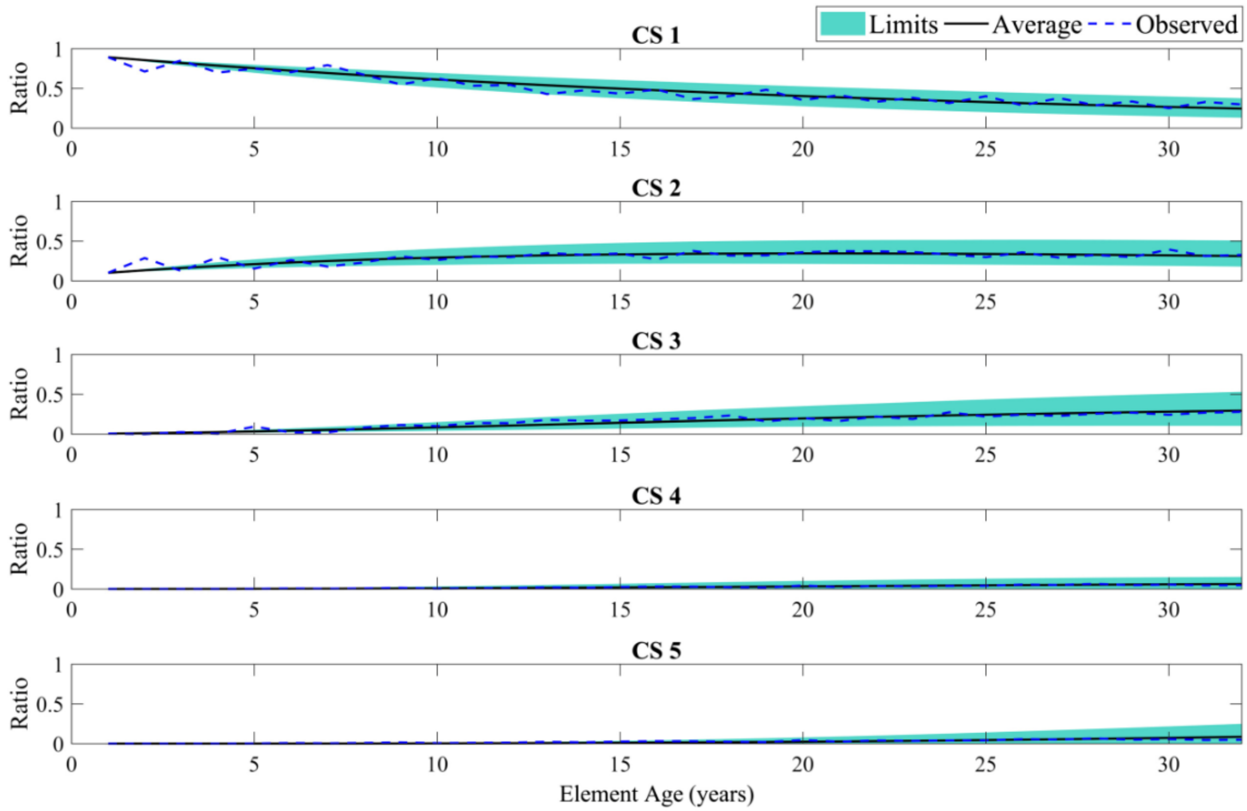

Fig. 6. Element 107 deterioration process simulation on a single element level

Element 12

The Monte Carlo simulation of Element 12 is presented in Fig. 7. The mean of the simulation is consistent with the observed deterioration process for each condition state. In particular, the mean of the simulations of condition states 3, 4, and 5 closely align with the observations. The observed deterioration processes of condition states 1 and 2 are bouncing around the mean of the simulations at the beginning of the study period. In the later period, the mean of the simulations is well-matched with the observations, especially after age 25. Similar to the simulation of Element 107, the gray bands represent the uncertainty of the deterioration process simulations. The condition state observations are generally covered by gray bands.

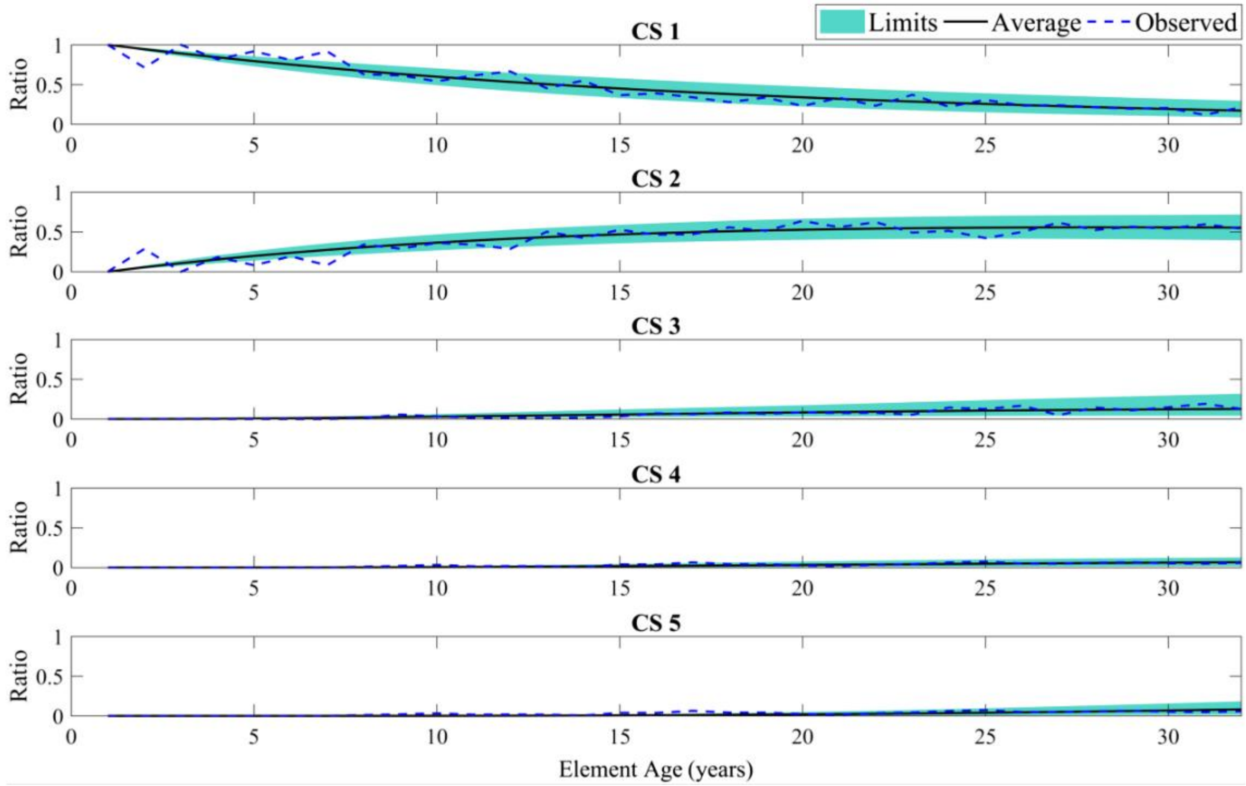

Fig. 7. Element 12 deterioration process simulation on a single element level

Deterioration Process Simulation on a System Level

The deterioration process of the bridge element system in this example application is simulated using the proposed method on a system level. The procedure in this case is

Step 1. Generate 5,000 deterioration process instances for Element 301 and 313 using the proposed method on a single element level.

Step 2. Use the subordinate deterioration model to compute 5,000 deterioration process instances for Element 311 corresponding to the instances of Element 301.

Step 3. Generate 5,000 deterioration process instances of Element 107 based on the instances of Element 301, 311, and 313 using the subordinate deterioration model.

Step 4. Calculate 5,000 possible deterioration process instances of Element 12 based on the simulation of Element 107.

In this process, 5,000 deterioration process instances of this bridge element system were generated. The results of Elements 107 and 12 are provided and compared with the observed deterioration processes in Fig. 8. There are two major differences between the simulation on the single element level and the system level. First, the mean of the simulations on the system level is slightly closer to the observed deterioration processes in general, although this is not obvious in Fig. 8. Later in this section, a comparison between the *RMSE* of simulations on the single element level and system level are provided to demonstrate this point. Second, the improvement of the model's performance is more significant at the end of the simulation period, which means that the model on the system level is more reliable in simulating the long term structure deterioration process.

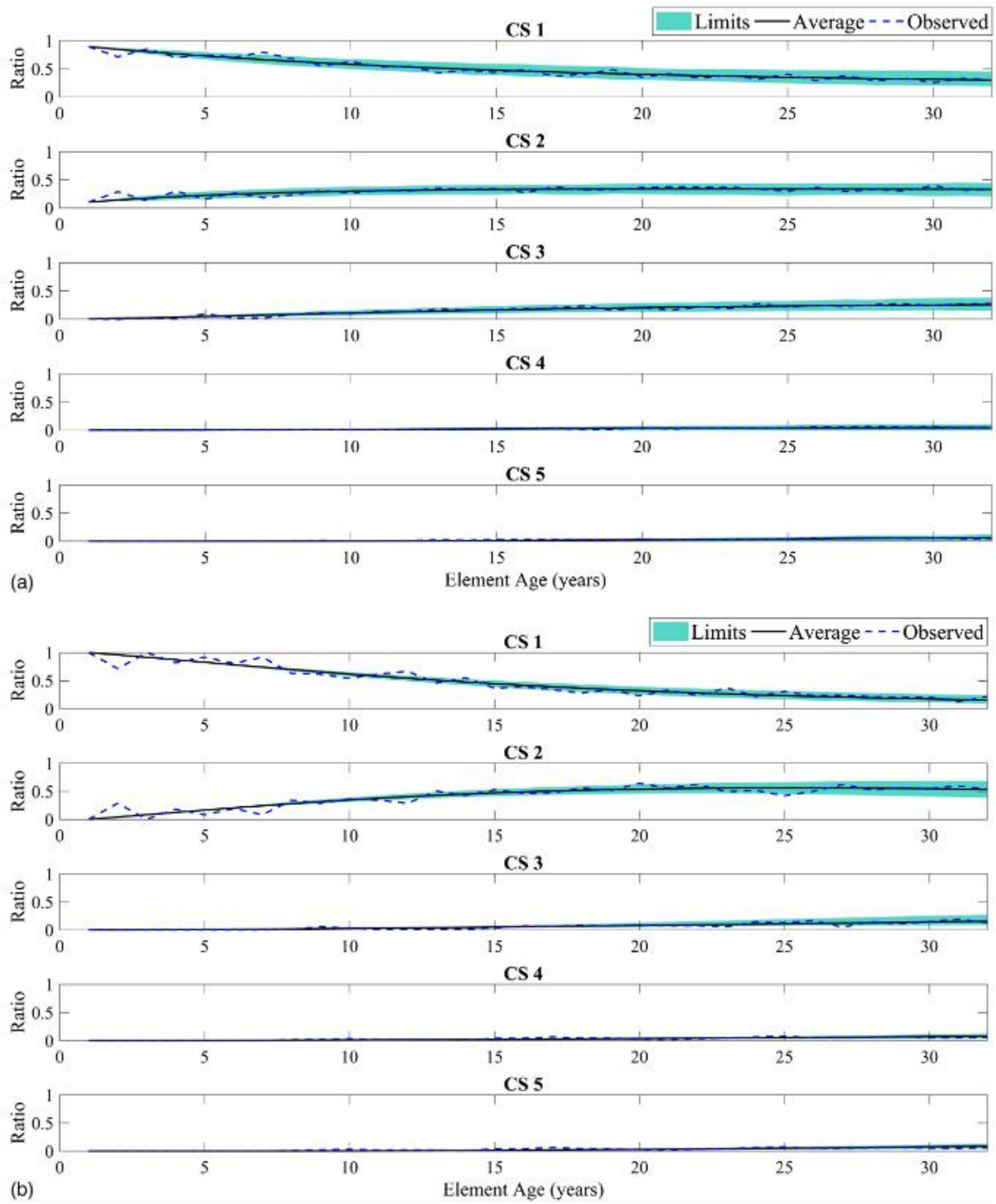

Fig. 8. Deterioration process simulation on a system level (a) Element 107 and (b) Element 12

The RMSE between the mean of the simulation and the observations was calculated to evaluate the accurary of the proposed method. The *RMSE* is given by

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(\widehat{CS}_i - CS_i\right)^2}{N}} \quad (9)$$

where $\widehat{CS}_i$ is the mean of the condition state simulation at age i, CS_i is the observed condition state at age i, and N is the length of the simulation period. The *RMSEs* for each condition state are calculated for the simulation on both the single element level and the system level. The results are shown in Table 4. For Element 107, the *RMSEs* for both single element and bridge element system are less than 0.07, which means both simulations fit well with the observations. Similarly, for Element 12, the results for both situations are fairly accurate compared to the observation because of the small *RMSE* (less than 0.09). The *RMSE* for simulations on a system level are smaller than that on a single element level except for the condition state 4 of Element 107. The simulation of condition states 3 and 5 of the Element 107 improved significantly by using the proposed method on the system level, while only a slight improvement resulted for the condition state 1. For condition states 2 and 4 of Element 107, the difference between the *RMSEs* on both situations was very small. The *RMSE* for each condition state of Element 12 was smaller on the system level compared to that on the single element level.

Table 4. *RMSE* Between the Bridge Element Deterioration Process Simulations and Observations on Both the Single Element and System Level for the Training Bridge Set

Condition States	**Element 107**			**Element 12**		
	Single	**System**	***Diff*** **(%)**	**Single**	**System**	***Diff*** **(%)**
CS 1	0.0592	0.0534	9.6	0.0829	0.0798	3.7
CS 2	0.0464	0.0462	0.2	0.0796	0.0766	3.8
CS 3	0.0315	0.0254	19.4	0.0275	0.0254	7.6
CS 4	0.0065	0.0067	-4.6	0.0138	0.0132	4.3
CS 5	0.0131	0.0088	33.6	0.0190	0.0165	13.2

Note: $Diff = (RMSE_{Single} - RMSE_{System})/RMSE_{Single} \times 100\%$

Model Evaluation

The inspection record of the testing bridge set is used to evaluate the performance of the proposed model. Starting with known initial condition state of the testing bridge set, the proposed method is performed over the entire study period. The result is compared with the observation of the testing bridge set. Here, same as the previous sections, the results of Elements 12 and 107 are provided as a demonstration.

Fig. 9 and 10 show the comparison between the condition state simulations and observations for Elements 107 and 12, respectively. For the deterioration process of Element 107 simulated on the single element level, the simulation captured the overall trend of the actual deterioration process. The mean of condition state 2 simulation is slightly overestimated, and the mean of condition states 1, 4, and 5 simulation are slightly underestimated. For Element 107 on system level, the mean of the simulation is a better match with the observation, esepcially, after age 15. For Element 12, the simulations for both situations have a good fit with the observed condition state. The accuracy of Element 12 condition state simulation on the system level is higher than that on the single element.

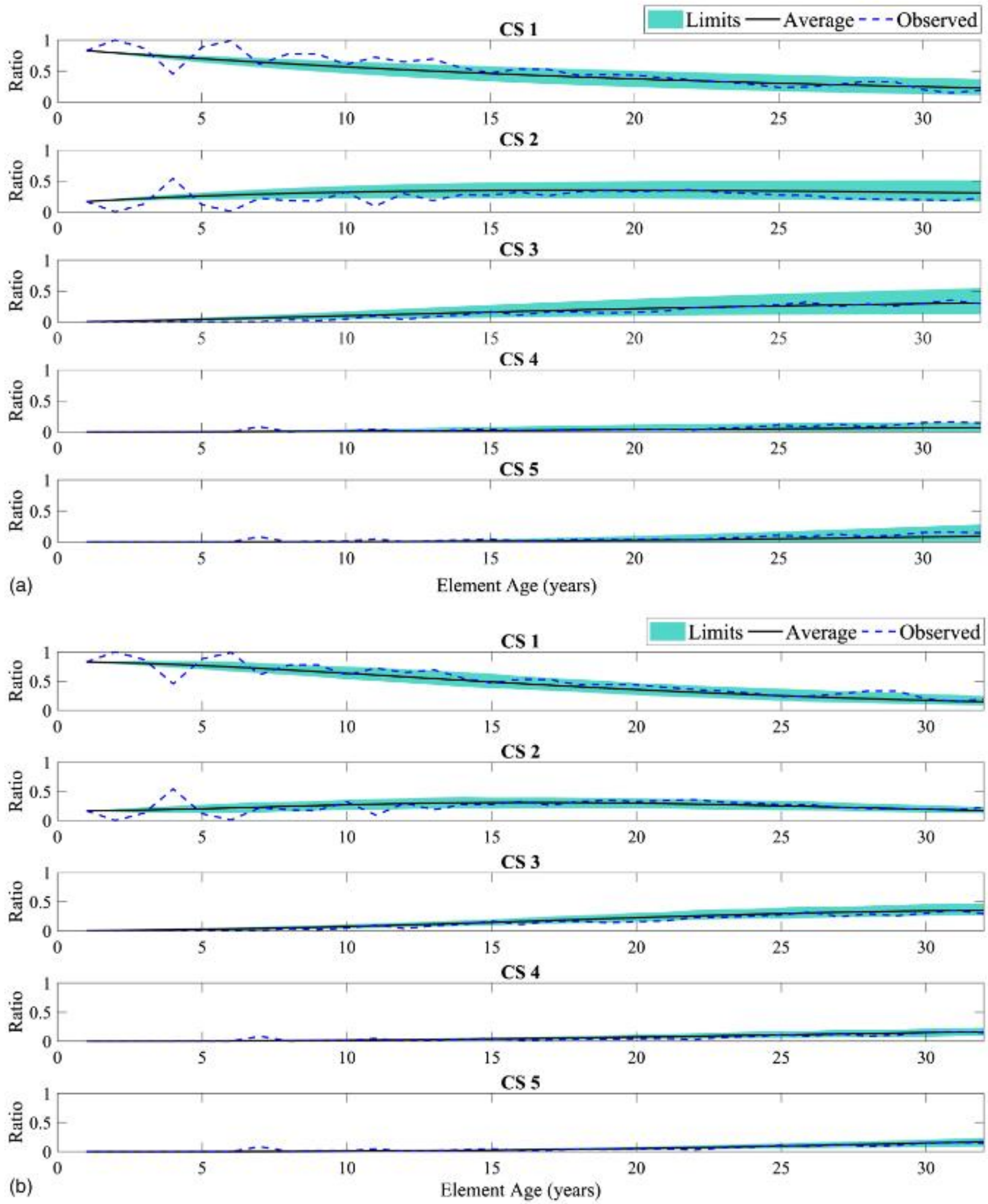

Fig. 9. Element 107 deterioration process simulations versus observations for testing bridge set on (a) single element level and (b) system level

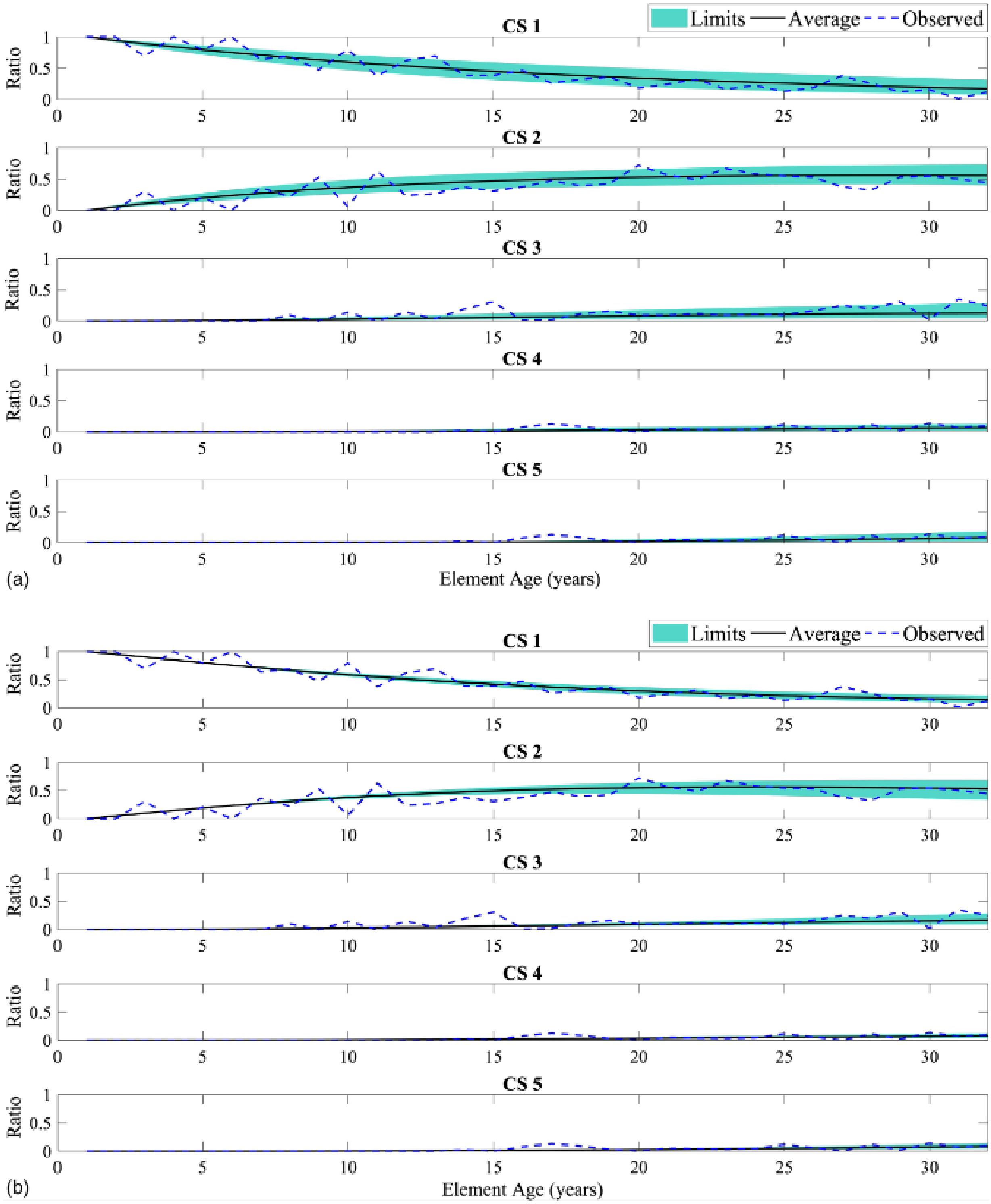

Fig. 10. Element 12 deterioration process simulation versus observation for testing bridge set on (a) single element level and (b) system level

The *RMSE* between the deterioration process simulations and observations was calculated for Elements 107 and 12 on both the single element level and system level (Table 5). For both elements, the *RMSE* on the system level simulation are generally smaller than that on the single element level, except for the condition state 3 of Element 107 and the condition state 2 of Element 12. For Element 107, the accurary of the simulation of condition states 1, 2, 4, and 5 have a significant improvement when using the proposed method on the system level. For condition state 3 of Element 107, the difference between *RMSE* on the single element level and the system level are is almost negligible. The condition states 1, 3, 4, and 5 simulation of Element 12 have a slightly higher accurary when using the proposed method on the system level.

Table 5. *RMSE* Between the Bridge Element Deterioration Process Simulations and Observations on Both the Single Element and System Level for the Testing Bridge Set

Condition States	Element 107			Element 12		
	Single	System	*Diff* (%)	Single	System	*Diff* (%)
CS 1	0.1215	0.1087	10.5	0.1185	0.1129	4.7
CS 2	0.1195	0.0951	20.4	0.1381	0.1410	-2.1
CS 3	0.0399	0.0402	-0.8	0.0905	0.0855	5.5
CS 4	0.0386	0.0251	35.0	0.0351	0.0332	5.4
CS 5	0.0324	0.0215	33.6	0.0366	0.0343	6.3

Note: $Diff = (RMSE_{Individual} - RMSE_{System})/RMSE_{Individual} \times 100\%$

CONCLUSIONS

The primary objective of this research is to develop a method for simulating the deterioration process of civil infrastructure on a system level while also analyzing the uncertainties of the simulation. The approach uses a method based on the age of the infrastructure elements to calculate the condition state distribution. Bayesian MCMC is used to drive the probability distributions of the Markov Chain transition probabilities of elements being studied. The Monte Carlo simulation is then applied to generate a large number of deterioration process instances. The uncertainties of the deterioration process simulations are analyzed based on these instances. A Markov Chain-based method is modified to calculate the deterioration process that considers the interaction between multiple elements. As a demonstration, the method is applied to a bridge element system from the VDOT bridge inspection database. In the example application, the deterioration processes on the single bridge element level and the system level were simulated and compared.

The main benefit of the proposed method is that it is capable of simulating the deterioration processes of civil infrastructure on a system level while also providing a measure of the uncertainty of the predictions. In addition, the proposed method is more straightforward to implement within current IMS compared to other methods, such as neural networks and case-based reasoning models. This is because the proposed method is built on a stochastic model, which has been shown to provide better extrapolation capabilities and has been widely used in current IMS to make effective and efficient MR&R strategies. All parameters used in this method are calibrated using historical inspection records, an approach which avoids the subjectivity of assigning these parameters based on engineering judgment. Furthermore, the uncertainty of deterioration process, which is usually ignored by previous models, is considered in the proposed method. An uncertainty analysis of the deterioration process provides vital information upon which decision makers to make effective MR&R judgements.

With the interaction between structure elements being considered, the proposed method performs better at estimating deterioration processes compared to methods that ignore element interactions. The accuracy of the proposed method has 4% to 30% improvement when additional information about the condition state of interacting elements is considered in the calculation. The higher accuracy in predicting infrastructure's future condition state is important for making optimal MR&R decisions under financial constraints.

Three approaches for further advancing this work are (1) using a more realistic stochastic model instead of Markov Chain, (2) testing the model on different types of civil infrastructure and more complex systems, and (3) developing a SDM with less parameters to make sure the uncertainty can be thoroughly considered during the simulation. Markov Chain ignores the effect of sojourn time, i.e., the time spent in one condition state before transitioning to another. The semi-Markov Chain can be applied to address this limitation. In this study, a simple bridge element subsystem is tested as a demonstration. A more complex bridge element subsystem or other civil infrastructure systems, such as buried pipeline systems and pavements, can be tested in future research to verify the feasibility and accuracy of the proposed model.

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SUPPLEMENTAL DATA

Figs. S1-S4 are available online in the ASCE Library (https://ascelibrary.org).

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SUPPLEMENTAL DATA

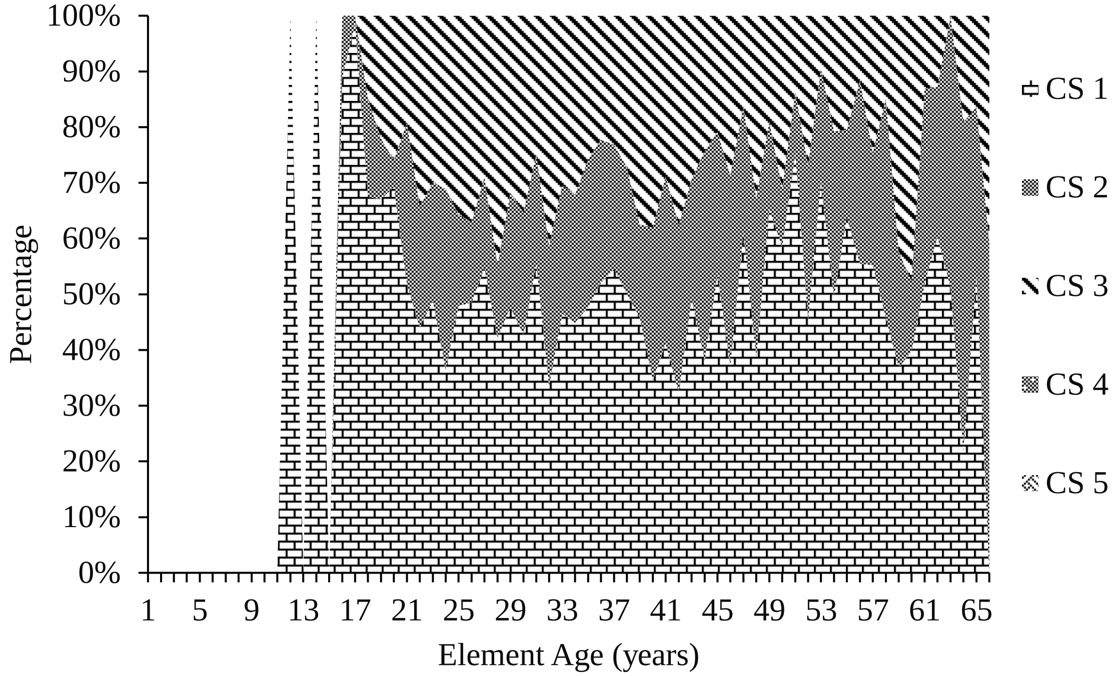
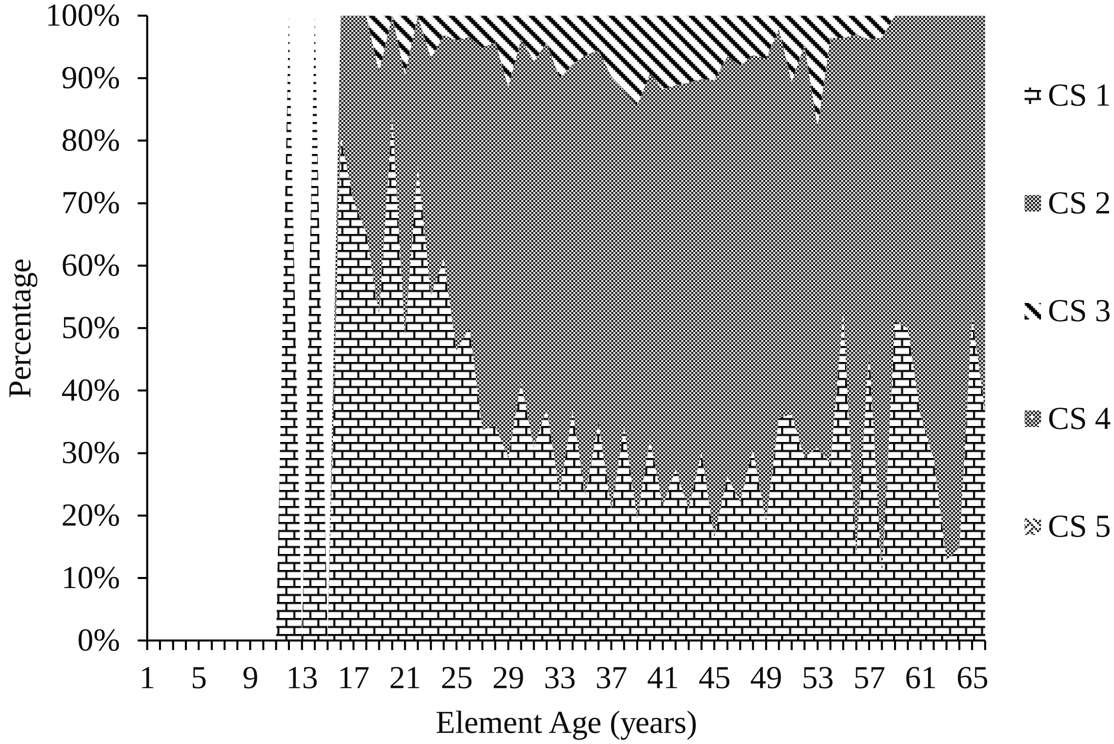


Fig. S1. Condition State Distribution of Element 301 on the Training Bridge Data Set



776

777 **Fig. S2.** Condition State Distribution of Element 311 on the Training Bridge Data Set

778

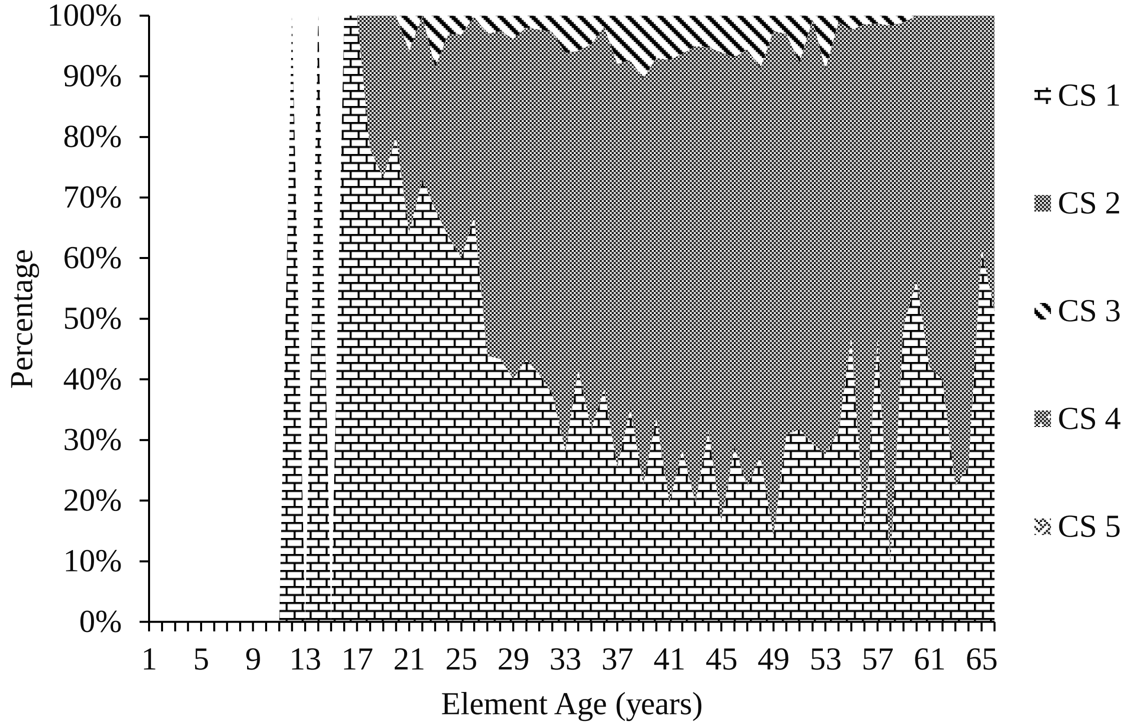


Fig. S3. Condition State Distribution of Element 313 on the Training Bridge Data Set

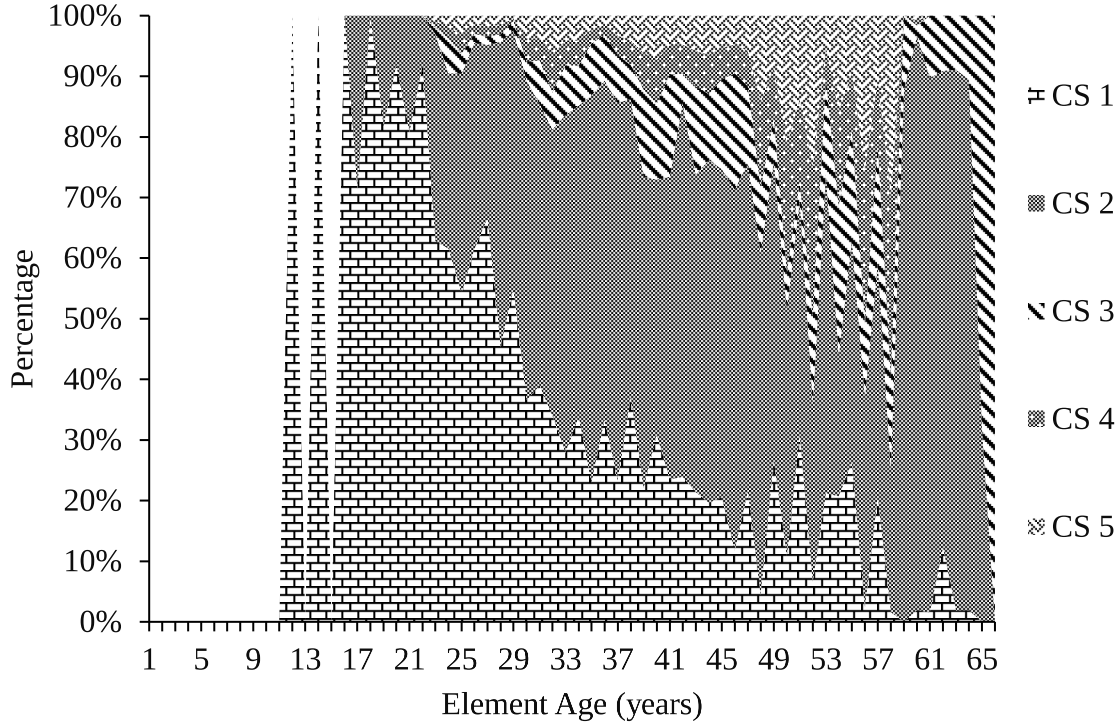


Fig. S4. Condition State Distribution of Element 12 on the Training Bridge Data Set