A hierarchical network-based algorithm for multi-scale watershed delineation

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Abstract

Watershed delineation is a process for defining a land area that contributes surface water flow to a single outlet point. It is a commonly used in water resources analysis to define the domain in which hydrologic process calculations are applied. There has been a growing effort over the past decade to improve surface elevation measurements in the U.S., which has had a significant impact on the accuracy of hydrologic calculations. Traditional watershed processing on these elevation rasters, however, becomes more burdensome as data resolution increases. As a result, processing of these datasets can be troublesome on standard desktop computers. This challenge has resulted in numerous works that aim to provide high performance computing solutions to large data, high resolution data, or both. This work proposes an efficient watershed delineation algorithm for use in desktop computing environments that leverages existing data, U.S. Geological Survey (USGS) National Hydrography Dataset Plus (NHD+), and open source software tools to construct watershed boundaries. This approach makes use of U.S. national-level hydrography data that has been precomputed using raster processing algorithms coupled with quality control routines. Our approach uses carefully arranged data and mathematical graph theory to traverse river networks and identify catchment boundaries. We demonstrate this new watershed delineation technique, compare its accuracy with traditional algorithms that derive watershed solely from digital elevation models, and then extend our approach to address subwatershed delineation. Our findings suggest that the open-source hierarchical network-based delineation procedure presented in the work is a promising approach to watershed delineation that can be used summarize publicly available datasets for hydrologic model input pre-processing. Through our analysis, we explore the benefits of reusing the NHD+ datasets for watershed delineation, and find that the our technique offers greater flexibility and extendability than traditional raster algorithms.

Keywords: Geographic Information Systems; Geographic Information Science; Terrain Analysis; Hydrologic Analysis; Spatial Analysis

1 1. Introduction

A watershed boundary defines the land surface that contributes streamflow to a single outlet location (Chow *et al.*, 1988). With advancements in geospatial software and readily available remotely sensed data, geographic information system (GIS) analysis have become widely used by hydrologists for determining a watershed boundary. Many research studies have investigated the various terrain processing components of GIS watershed delineation, such as methods for surface smoothing (Hutchinson, 1989), determination of flow direction (Douglas, 1986), slope and aspect calculations (Hodgson, 1998), depression filling (Jenson and Trautwein, 1987), and the extraction of drainage channels (O'Callaghan and Mark, 1984). These are only a few examples of the research that helped shape this domain; Moore *et al.* (2006) offers a more complete summary of the field.

The advent of high resolution digital terrain data and the need to analyze larger watersheds for envi-11 ronmental policy have resulted in efforts to advance the computational efficiency of terrain processing for 12 hydrology applications. Recent studies have employed high performance computing (HPC) environments to 13 overcome such computational limitations (Mineter, 2003; Wang and Armstrong, 2009; Huang et al., 2011). 14 Through these studies it has been demonstrated that HPC solutions have the potential for large performance 15 gains by uncovering the intrinsic parallelism in traditional geospatial algorithms (e.g. Wang and Armstrong, 16 2009). Parallel algorithms operate by sharing the computational burden of data processing with multiple 17 resources, and communicating data among each other using protocols such as the Message Passing Interface 18 (MPI) (Xie, 2012). These approaches use advanced computational algorithms for delineating watersheds 19 from digital elevation models (DEMs), mostly using the divide and conquer approach (Hutchinson et al., 20 1996). 21

A similar, albeit fundamentally different approach for processing large datasets, is to leverage idle com-22 puting power by means of high throughput computing (HTC). HTC is a method for flexible distributed 23 computing that takes advantage of relatively inexpensive collections of computing resources to achieve per-24 formance gains comparable to large HPCs (Thain et al., 2005). It is a convenient solution for processing large 25 amounts of data that enables organizations to take advantage of existing network compute power without 26 the need for special computer hardware. The goal is to achieve speedup over longer periods of time using 27 computing grids rather than emphasizing computer architecture (Chaudhry et al., 2005). Recent studies 28 have shown that this approach is effective in achieving significant computational speedup when processing 29 large raster datasets (Gong and Xie, 2009; Huang and Yang, 2011). 30

While these approaches have been used extensively to processes large datasets, they require access to advanced computing techniques and resources. For instance, a great deal of expertise is required to design

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and use parallel HPC software modules because of their inherently high "learning curve," which has a 33 tendency to deter both commercial and academic developers (Mineter et al., 2000; Lu et al., 2010). An 34 exception to this is are software that have adapted their algorithms to distribute computational load among 35 processor threads to incorporate some of the HPC advantages (i.s. distributed computing) on desktop 36 computers. TauDEM is one software application that employs this tactic to provide users with the best 37 of both worlds (Wallis et al., 2009). Similarly, HTC requires a large network of idle computers as well 38 as specialized scheduling software to balance computing load across the network. Overall HPC and HTC 39 solutions can be effective for data intensive computations, however they require specific computer hardware 40 and a high level of sophistication. Moreover, many water resources professionals still rely on desktop 41 computing environments as their main platform for watershed analysis. We lack a versatile approach of 42 watershed delineation capable of efficiently resolving a wide range of spatial scales, without the use of HPC, 43 HTC, or similar computing environments.. 44

An alternative strategy for watershed delineation is to rely on pre-processed vector data. One example of 45 this approach was presented by Djokic and Ye (2000), which aimed to overcome the computationally intensive 46 nature of watershed delineation by separating static terrain-based properties from the delineation procedure. 47 They proposed that since terrain measurements do not change often, they should not be linked directly to the 48 delineation procedure. Rather, catchment geometries are processed prior to watershed delineation and later 49 leveraged to construct a watershed boundaries. The major contribution of this work was their methodology, 50 Fast Watershed Delineation (FWD), which is capable of rapidly yielding watershed boundaries using only 51 desktop computing resources. Several additional efforts have been made to extend this technique for serving 52 watershed delineations via web services. For example, the ArcGIS Watershed Delineation service provides a 53 quick method for retrieving watershed delineations (Kopp et al., 2013). Both of these approaches, however, 54 require that computationally intensive catchment pre-processing routines have been completed prior to 55 usage. Similar web based efforts have been made by the Environmental Protection Agency (EPA) and 56 United States Geologic Survey (USGS) to produce the Navigation Delineation Service and StreamStats, 57 respectively. The EPA Navigation Delineation Service leverages the NHD+ dataset to determine watershed 58 boundaries and has been implemented by the Consortium of Universities for the Advancement of Hydrologic 59 Science, Inc. (CUAHSI) HydroDesktop software, to delineate watershed boundaries which are then used to 60 search for observation data within the Hydrologic Information System (Ames et al., 2012). Similarly, the 61 USGS StreamStats application offers a delineation service that is built using the NHD+ dataset and ArcGIS 62 tools, but it also requires significant pre-processing (Guthrie et al., 2009; Ries et al., 2009). 63

Since the work of Djokic and Ye (2000), new datasets have become available such as the USGS National Hydrography Dataset Plus (NHD+). The NHD+ is a dataset derived from measured elevation, digitized hydrography, and the USGS Watershed Boundary Dataset (WBD) to accurately match known surface hydrology. While the NHD+ contains elevation derived products such as flow direction and flow accumulation

grids for the entire U.S., it also provides pre-processed hydrologic catchment boundaries and river flow net-68 works. These can be leveraged to rapidly delineate watershed boundaries while eliminating data intensive pre-processing routines. Our approach is to leverage the concepts outlined by Djokic and Ye (2000), and the 70 pre-processed NHD+ data to reconstruct watershed boundaries from pre-computed catchment geometries... 71 Using graphing algorithms, upstream flow direction cells and ultimately catchment boundaries are identified 72 for a given outlet location. We demonstrate how our approach is capable of rapidly yielding watershed 73 boundaries for large areas on a desktop computer, while also delineating small catchments in a timely man-74 ner. It is then applied to the delineation of subwatersheds to demonstrate how it can be adapted for other 75 common hydrologic tasks. Overall we demonstrate how our approach is a versatile solution for performing 76 multi-scale watershed delineation on a desktop computer. 77

78 2. Method

Our method for watershed delineation is a two-step approach that borrows from graph theory to trans-79 form river flow attributes and known watershed surface runoff patterns into relational networks. While 80 hydraulic river flows are used to identify fluxes between catchments, surface runoff is used to establish flow 81 paths between raster cells. Furthermore, the hydraulic river flow graph is used to determine the "upper" 82 portion of the watershed, and in contrast the surface runoff graph is used to determine the "lower" portion 83 of the watershed. These upper and lower geometries are later combined to form the complete watershed 84 boundary. This delineation technique requires both reach network and catchment input shapefiles, and 85 relies on the specific relationships that can be established between them. More specifically, each feature in 86 the reach network must have defined start and end nodes which are associated with other reaches in the 87 network, as well as attributes that support network traversal. In addition, each reach must be associated with a single catchment geometry. This section explains how the NHD+ dataset can be leveraged to rapidly 89 delineate watersheds, however as long as the aforementioned constraints are met the same technique can be 90 applied to other datasets. 91

Our method uses graph theory to form relationships between the hydraulic characteristics of reaches as 92 well as gravity driven flows among terrain grid cells. The basic concept is that a graph consist of independent 93 entities (i.e. vertices) that are related to one another through connections (i.e. edges) (Marcus, 2008). This 94 basic concept is illustrated in Figure 1. Once a graph is assembled, algorithms are used to navigate and 95 traverse the nodes to solve a variety of problems, such as the famous traveling salesman problem (Hagberg 96 et al., 2008). There are also many different types of graphs, such as undirected, directed, multigraphs, etc., 97 which can be applied to solve problems in nearly every discipline (Rosen, 2003). In our work we use the 98 NHD+ catchments to define graph vertices and the NHD+ reaches as graph edges between these nodes. 99 The terrain is also represented as a graph, where the centroid of each cell in a DEM is a vertex and the flow 100



Figure 1: A basic directional graph consists of vertices that are connected by edges to describe the relationship among them.

direction at the node is used to establish an edge with its downstream neighbor. These two directed graphs enable us to traverse the NHD+ dataset in a hydraulically upstream/downstream manner. Both of these concepts are further explained in the following paragraphs.

First, consider that any given watershed may consist of one or more NHD+ catchments. Figure (2, i) 104 shows the NHD+ river network overlaying a small watershed consisting of pre-delineated catchments. These 105 catchments are related to one another by river flow attributes, for instance, each catchment drains into 106 exactly one of its neighbors. Since, each river in the NHD+ dataset is associated with an upstream and 107 downstream reach, this information can be used to create the graph illustrated in Figure (2, ii). This graph 108 network defines the water flow paths among catchments. Using our approach, we assume that a watershed 109 is encapsulated within this network and moreover consists of one or more catchments that can be identified 110 by hydraulic river flow attributes. In this manner, all catchments upstream of a given outlet location can be 111 quickly determined (Figure 2, iii), and subsequently merged together to resolve the geometry for the upper 112 portion of the watershed (Figure 2, iv). 113

A watershed outlet can be located anywhere within a catchment, not necessarily coinciding with the drainage point used in the NHD+. Therefore, we must consider an alternative approach for resolving the remaining, lower portion, of the watershed. This is accomplished by leveraging watershed surface flow attributes. First, a mathematical graph is created using flow direction raster cell values. In a single-flowdirection raster grid, each pixel contains a numeric value that defines the direction water flows from the



Figure 2: A graphical representation of the watershed delineation procedure.

surface in flooded conditions. By iterating through these cell values, a graph is constructed whereby each 119 node represents the centroid of a cell and each cell is connected to its "downstream" neighbor. Figure 2, 120 v) illustrates how an NHD+ catchment is transformed in to a dense graph network of cell-level flow paths. 121 Using graph theory, all of the raster cells contributing to a given location, i.e. watershed outlet, can be 122 determined by tracing the flow network in the upstream direction. This task is made trivial by leveraging 123 well-established software libraries that employ optimized graph traversing algorithms. Once the upstream 124 graph elements are known, the boundary for the lower geometry can be constructed by eliminating all 125 interior graph nodes (Figure 2, vi). Once the upper and lower portions of the watershed have been resolved 126 (Figure 2, vii), they are spatially merged together using GIS software to produce the complete watershed 127 boundary (Figure 2, viii). 128

To construct the catchment and flow direction graphs, existing software libraries can be leveraged such 129 as the NetworkX Python library (Hagberg et al., 2008). Using the aforementioned approach for constructing 130 flow direction graphs, an edge is created between each cell and its "downstream" neighbor. However, applying 131 this methodology for large areas results in extremely large graphs, and is therefore infeasible. For instance, 132 creating a graph network consisting of 1 arcsecond data covering South Carolina, results in approximately 133 8.88×10^7 graph elements. A graph of this magnitude is impractical because its memory footprint is too large 134 for most desktop computers. Since this graph is only used to resolve the lower portion of the watershed, it 135 must only cover the area of an NHD+ catchment. Given this, single flow direction rasters can be extracted 136 individually for each NHD+ catchment and graphs can be subsequently created. Moreover, these catchment 137 level graphs can be serialized and stored for later use to further eliminate redundant operations. This 138 technique results in one graph for each NHD+ catchment boundary. In contrast, the catchment graph is 139 much less dense and as a result a single graph will suffice for an area covering South Carolina. By using this 140 graphical approach, (1) all upstream catchments are identified, (2) the raster cells contributing flow to the 141 outlet can be determined, and finally (3) the results of these operations can be merged to form a complete 142 watershed. 143

This method can also be extended to support the delineation of subwatersheds, which is an important feature of GIS software for hydrology applications. Subwatersheds are generally derived from outlets that correspond with available observation data, and are often used to pre-process or summarize watershed related data for model inputs. A similar approach is taken to determine these subwatershed areas, albeit with a small modification. First all upstream catchments are determined. Next, the catchments corresponding to each individual outlet are isolated, such that each catchment is associated with only one outlet location. For each outlet, the upper catchments and lower catchment are combined in a manner consistent with Figure 2.

¹⁵¹ 3. Implementation

The NHD+ provides many GIS data products to the public for free. The watershed delineation technique 152 presented in this work uses several of these data products, as well as supplementary database files used to 153 enhance their geospatial representations. While a newer version of the NHD+ dataset (version 2) is currently 154 available, this work was initiated and completed using the NHD+ version 1. These data provide additional 155 feature-based values and attributes to support the NHD+ vector data. This supplemental information 156 can be leveraged to establish relationships between features of multiple NHD+ vector files. Moreover, 157 the delineation algorithm relies on the NHD+ vector products and these additional datasets to establish 158 relationships between digitized rivers and catchment boundaries to rapidly delineate watershed boundaries. 159 There exists a many-to-one relationship between the NHD+ river and catchment features. This means 160 that there is at least one reach for every catchment defined in the dataset. As a result, the NHD+ reaches 161



Figure 3: Joining the NHDFlowLineVaa dataset to both the NHDFlowLines and Catchment shapefiles provides the necessary attributes to form a connection between river reaches and catchments.

can be used to identify specific catchments, but to do this additional data attributes must be appended 162 to the reach dataset. Figure 3 illustrates how the NHDFlowlineVAA.dbf (i.e. Value Added Attributes, 163 VAA) data product can be used to enhance the hhdflowline and catchment vector files. By spatially joining 164 the VAA attributes to both of these datasets, additional information is appended to each feature such as 165 from node, to node, hydroseq, and terminalpa. The from node and to node attributes are unique identification 166 numbers that denote the start and end nodes for every reach in the dataset. The hydroseq parameter is a 167 unique hydrologic sequence number assigned to each reach in the dataset. These sequence identifiers are 168 assigned such that upstream reaches have larger values than downstream reaches. Finally, the terminalpa 169 parameter defines the hydrologic sequence number of the terminal feature of the reach network. This data 170 is used to create a directed graph which can then be used to identify upstream or downstream reaches from 171 any location within the network. 172

Using this upstream and downstream reach information, a graphical network is created to represent 173 relationships between digitized reaches, and ultimately the catchment features. First, NHD+ reach and 174 attribute data is transformed into a graphical network which later provides a mechanism for tracing flow 175 path's and identifying upstream reaches id's using graph tracing algorithms. Figure 4 illustrates how this 176 graph network is created using the Python programming language and the NetworkX library (Hagberg et al., 177 2008). For each feature in the NHD+ reach network, a graph edge is established between its fromnode and 178 tonode identifiers. In addition, hydroseq and terminalpa values are stored as attributes on the graph object. 179 Once this process is complete for every reach feature, the graph is serialized for later use. Serialization is 180 an important part of this procedure because it enables the graph data object to be reloaded when needed, 181 rather than reconstructing it from scratch, which would be timely and inefficient. Using this graph, data for 182



Figure 4: Flow chart illustrating how NHD+ digitized reaches are transformed into a mathematical graph network that is then serialized for later use.

all elements preceding a given node can be identified using NetworkX graph tracing functions. This provides
an effective means for selecting the NHD+ reach elements that contribute flow to a common downstream
location. For example, given a watershed outlet, all upstream reach attributes can be quickly identified.
These data are then used to select the corresponding NHD+ catchment geometries.

Databases offer a mechanism for archiving large amounts of data in an easily accessible manner. Because 187 of this, a database was chosen for storing the NHD+ catchment geometries and feature attributes. For 188 this implementation the open source PostgreSQL database was chosen because it can easily be extended 189 to support spatial data queries using PostGIS. This setup enables spatial data to be archived in an easily 190 accessible format as well as retrieved using spatial data querying using standard SQL statements. Moreover, 191 the PostGIS extension is equipped with numerous spatial operations that can be performed on-the-fly when 192 querying data. Therefore, the NHD+ catchment geometry data was loaded into a PostgreSQL database 193 along with the appended NHD+ value added attributes. In addition, an empty network database field 194 was defined to store cell-level NetworkX graphs which are constructed on-the-fly using flow direction values 195 within the selected catchment boundary. For instance, when an outlet is chosen and the corresponding 196 catchment is identified, we must first check to see if a cell-level graph network exists in the database for this 197 catchment. If not, it is created on-the-fly and saved in the database for later use. This design consideration 198 aims to reduce unnecessary pre-processing steps, however this calculation can be performed ahead of time 199 if maximum speed is a priority (e.g. web deployment). Once these catchment attributes are loaded into the 200 database, spatial SQL queries are used to quickly extract catchment geometries when needed. 201

With the pre-processed NHD+ catchment data stored in a spatial database, features are queried using 202 standard and spatial SQL statements. For instance, all catchments "upstream" of a graph location have 203 a hydroseq identifier greater than that of the current location yet no greater than the largest hydroseq 204 in the graph tree. The upper limit of this range is quantified as maxseq, and is calculated by simply 205 iterating through all "upstream" nodes in the graph. Furthermore, all features must also belong to the same 206 terminalpa. Therefore, given an outlet location as outlet_node, all upstream catchments can be identified 207 in a manner consistent with Figure 5. This results in the extraction of all NHD+ catchment geometries 208 upstream of the outlet. These geometries are deserialized and subsequently merged together by performing 209 a spatial union, which results in the upper portion of the watershed boundary. Since the watershed outlet 210 can be located anywhere within the lowest catchment of the watershed, an alternative approach must be 211 used to resolve the lower portion of the watershed boundary. 212

Since NHD+ catchment areas are relatively small in scale, mathematical graphs can be created to represent the flow paths between the interior flow direction values for each catchment. To create one of these networks, the NHD+ flow direction grid is first extracted over the area of a single catchment geometry. For each cell within this smaller grid, graph edges are created between each cell and its "downstream" neighbor. The "downstream" neighbor is easily identified for each cell using the single flow direction notation (i.e. D8

```
SELECT Geometry
FROM catchments_datatable
WHERE HYDROSEQ > outlet_node.HYDROSEQ
AND HYDROSEQ <= MaxSEQ
AND TERMINALPA == outlet_node.TERMINALPA</pre>
```

Figure 5: SQL query for extracting catchment geometries stored in a PostreSQL database using HYDROSEQ and TERMINALPA attributes.

grid values). The final product is a graphical network consisting of edges that define the flow paths between raster cells, which are confined to a single NHD+ catchment boundary (i.e. catchment flow path graph). As mentioned earlier, this operation is performed on-the-fly when needed, and stored in the spatial database within the *network* field as a serialized NetworkX object.

Using this flow direction graph, all nodes (i.e. cells) upstream of the outlet can be quickly identified using graph traversing algorithms. The result is a set of coordinates that represent all cell locations "upstream" of the outlet, but within the "most downstream" NHD+ catchment. By tracing the edge of this delineation upstream from the outlet, the border locations can be identified. Once this boundary is identified, a polygon object is created that represents the lower portion of the watershed. Finally, the upper and lower watershed polygons are combined to form a single watershed boundary. Once complete, the overall catchment boundary is saved as Well Known Text (WKT) and are later converted into Esri shapefile format for visualization.

229 4. Application

Two studies were conducted to evaluate the application of the provided watershed delineation technique. 230 First it is evaluated in its ability to delineate watersheds at various spatial scales, then it is applied to the 231 delineation of subwatersheds. While similar Three community accepted software applications are used to 232 provide context for the general accuracy of the hierarchical algorithm. The first benchmark software, Esri's 233 ArcGIS, is a widely used commercial-grade GIS suite. It consists of many tools for GIS analysis, including a 234 hydrology toolbox which is capable of performing a wide range of hydrology-related data processing routines. 235 In addition, ArcGIS contains a built-in Python interpreter, which enables these tools to be executed pro-236 grammatically. This functionality was leveraged to automate the ArcGIS watershed delineation procedure, 237 which consisted of executing several tools in series. 238

The second benchmark software, Terrain Analysis Using Digital Elevation Models (TauDEM), is an open source terrain analysis project (Tarboton *et al.*, 1997). It employs parallel computing concepts to divide large datasets into smaller subsets. Terrain processing is performed on each of these subsets simultaneously, and messages are passed between computational processes when necessary. Because of this design consideration, it can theoretically perform terrain processing on very large datasets at a much faster rate than other software. It has been used in a number of academic studies from basic watershed analysis (Tarboton *et al.*, 1997) to parallel terrain computations (Wallis *et al.*, 2009). Furthermore, the latest release of TauDEM (version 5.1) contains a toolbox plugin for ArcGIS (versions 9.3.1 – 10.0), therefore in a similar manner to ArcGIS, TauDEM tools can be executed autonomously through Python scripting. Our motivation for using these specific GIS software tools for comparison to our approach is to first provide context with regards to a closed-source commercially developed product, and then to an open-source academic tool.

The third benchmark software is ArcHydro Tools, which is a spatial processing tool pack that automates ArcGIS tools to perform advanced hydrology-related functions. In summary, it is an advanced information system that integrates the spatial and temporal aspects of hydrology to provide a complete GIS modeling and analysis suite (Maidment *ed.*, 2002). ArcHydro Tools is used in this work for its capacity to evaluate the accuracy and general efficiency of the provided delineation technique when applied to subwatershed delineation.

The testing all watershed delineations was performed using 32-bit Python interpreter on a desktop 256 computer with a quad core 2.80 GHz processor having 4 GB of memory. The execution of the provided 257 hierarchical delineation procedure is done by simply invoking the package from the commandline and passing 258 it either one or multiple outlet point coordinates. The user must be careful to supply outlets in the spatial 259 reference system that is used by the NHD+ catchment and river reach data. The algorithm uses several 260 scientific libraries such as NumPY, GDAL, and NetworkX which must be installed separately. In addition, 261 the NHD+ data must be downloaded, specifically the *catchments*, river reaches, and supplementary data 262 files. Lastly, for this work data is stored in a PostgreSQL database using the PostGIS extension for handling 263 spatial attributes. 264

265 4.1. Multi-Scale Watershed Delineation

To evaluate our approach for various watershed scales, five basins were delineated along the South 266 Carolina, Georgia border (i.e. the Savannah River basin) as well as one watershed that includes part of 267 North Carolina (i.e. the Cooper River basin), shown in Figure 6. All watershed delineations were performed 268 using input data provided by the NHD+ (i.e. flow accumulation and D8 flow direction), and the size of 269 the datasets used in each scenario are summarized in Table 1. While the input data for each delineation 270 scenario were provided at the same resolution, the grid sizes increased ascendingly in order to evaluate a 271 wide range of computational scales. The experiments began with smaller headwaters and progressed to the 272 larger downstream areas, concluding with the Savannah and Cooper River basins. 273

Table 1: General properties of the input raster datasets used for each watershed delineation scenario.

Dataset	Resolution	Dimensions	Grid Size	Disk Size
Scenario 1	$30 \mathrm{m}$	926×891	8.3×10^5	$3.15 \ \mathrm{MB}$
Scenario 2	$30 \mathrm{m}$	1835×2153	39.5×10^5	$15.07~\mathrm{MB}$
Scenario 3	$30 \mathrm{m}$	5822×5848	340.5×10^5	$129.88~\mathrm{MB}$
Scenario 4	$30 \mathrm{m}$	8287×7792	645.7×10^5	$246.32~\mathrm{MB}$
Scenario 5	$30 \mathrm{m}$	10165×11011	1119.3×10^5	$426.97~\mathrm{MB}$



Figure 6: Six watersheds were used to evaluate the performance of the hierarchical delineation approach. The Savannah River basin (Left) which was divided into five separate subwatersheds, and the Cooper River watershed (Right).

The watershed delineation procedure using the ArcGIS software suite is illustrated in Figure 7. First, the 274 processing environment is prepared for executing ArcGIS tools. This consists of loading Python modules as 275 well as registering ArcGIS extensions. Once the environment has been prepared, the input raster grids (i.e. 276 flow direction and flow accumulation) are reduced to the extent of the known watershed boundary using the 277 ArcGIS Clip function. In practice this extent is often unknown, but since this experiment is evaluating the 278 speed of ArcGIS watershed delineation, we assume that ideal input information is available. Next, a new 279 point shapefile is created that contains a single feature, the watershed outlet location. This outlet point is 280 then relocated to the neighboring raster pixel that has the highest flow accumulation value, using the Snap 281 Pour Point tool. This is done to ensure that the outlet resides at a location of high flow accumulation, 282

as determined by the terrain topography. Once these steps are complete, the Watershed tool is executed.

²⁸⁴ Finally, the raster output from the watershed delineation is converted to a Esri polygon shapefile.



Figure 7: The procedure used for watershed delineation using ArcGIS tools, automated using the Python programming language.

TauDEM watershed delineation requires similar pre-processing routines to the ArcGIS approach. Many 285 of the data pre-processing steps use standard ArcGIS tools, for instance Clip and Reclassify, whereas the 286 actual watershed delineation uses only TauDEM tools which are indicated by the blue bold headings in 287 Figure 8. While ArcMap was used to prepare the input data for this watershed delineation, alternative 288 software could also be used for this task without affecting the results. Figure 8 follows the delineation 28 procedure that is suggested by Tarboton (2010). First, a new point feature is created containing the desired 290 outlet location of the watershed. Next, the NHD+ flow accumulation, elevation, and flow direction grids 201 are clipped to the approximate extent of the watershed. While this step is optional, it can have a significant 292 effect on the overall speed of the delineation by reducing the size of the input data. Map algebra is used to 293 select flow accumulation cells based on a user specified threshold which creates a new raster grid in which 29 rivers have a value of 1 and all other cells have a value of 0. This new river grid is then used to snap the 295 outlet onto the river network using the TauDEM Move Outlets To Streams tool. This aligns the desired 296 outlet with the watershed outlet as defined by the terrain. Next, the clipped flow direction grid is converted 297 from the tradition single-flow-direction notation, into the single-flow-direction values used by TauDEM (i.e. 298 $\{1,2,3,4,5,6,7,8\}$). The Peuker Douglas Stream Definition tool is executed using the snapped outlet, and the 299 clipped flow accumulation, flow direction, and elevation grids as input. The tool creates several new output 300 datasets that summarize various reach properties. One in particular, the stream raster grid, identifies all of 301 the reaches upstream of the outlet location. This grid is used as input to the Stream Reach And Watershed 302 tool, along with the snapped outlet, and clipped flow direction, flow accumulation, and elevation grids. 303 Execution of this tool results in several more raster grids such as stream order, stream connectivity, stream 304

coordinates, and a watershed grid. The watershed grid is supplied as input into the Watershed Grid To shapefile tool which converts it into a vector file, thus completing the delineation procedure.



Figure 8: The procedure for watershed delineation using TauDEM tools, implemented using the Python programming language. The blue bold titles indicate operations performed by ArcGIS and bold titles indicate operations performed by TauDEM.

The provided approach and the two watershed processing techniques described above, were used to de-307 lineate six different watersheds. The objective of this study is to first evaluate the accuracy of hierarchical 308 watershed delineation approach, and second to quantify its performance. Using the ArcGIS and TauDEM 309 processing routines as benchmarks, we evaluate how well our approach performs with respect to commer-310 cial product and large scale terrain processing software. This experiment provides insight into the general 311 application of our watershed delineation approach compared with two widely used software suites. It re-312 vealed that there exist differences in the watershed boundaries computed by each software suite. Table 2 313 illustrates these discrepancies for each scenario as the percent difference of the area taken with respect to 314 the average computed watershed area. This serves as a basis of reference for comparing the variations in 315 the watersheds computed by each algorithm. Minor boundary differences exist between the TauDEM and 316 ArcGIS calculations, however these were found to be a result of polygon simplification. In contrast, the 317 provided delineation exhibits larger variations with respect to the average computed watershed areas. These 318 discrepancies may be explained by the manner in which the NHD+ dataset is created. For instance, the 319 NHD+ is constructed using modified DEMs that closely match the known digitized hydrography as well 320 as the WBD. The catchment features used in our approach were modified to match river streamflow and 321 velocity estimates (Johnston et al., 2009). In contrast, ArcGIS and TauDEM used surface elevation mea-322

Dataset	Average Area $\rm km^2$	ArcGIS % difference	TauDEM % difference	Hierarchical % difference
Scenario 1	336.893	0.543	0.538	1.081
Scenario 2	1716.599	0.124	0.123	0.248
Scenario 3	13841.155	0.029	0.029	0.058
Scenario 4	21581.296	0.018	0.019	0.037
Scenario 5	26774.317	0.018	0.018	0.037

³²³ surements which were not modified in the same manner. We suggest that the boundary differences outlined ³²⁴ in Table 2 are a byproduct of the NHD+ design and are therefore inherent to our technique.

Table 2: Differences in the watershed areas computed by each software suite. Variations are recorded relative to the mean watershed size for each scenario.

To provide context for the efficiency of our approach we compared the overall computation time for 325 each of the delineation approaches. In this analysis we only interested in the time it takes for a user to 326 perform a delineation from start to finish. We found that ArcGIS performs exceptionally well at small 327 scales, however it follows a non-linear trend as the size of the dataset continues to grow. This is expected 328 because the ArcGIS tools that are used in this study are designed for general purpose desktop computing 329 and are not designed for processing very large data sets . In contrast, TauDEM which has the capability of 330 processing large raster data, executed the fastest for all watersheds delineations except the largest. Similar to 331 ArcGIS, it also scales in a non-linear fashion as dataset size increases. However, this work was all performed 332 on a computer using 4 processing threads and 4GB of memory. We expect that TauDEM will perform 333 more favorably on a high performance computer. Our technique is slower when delineating watersheds 334 at small scales, however it follows a linear trend as area increases, and completes the largest delineations 335 significantly faster than the other software systems. For instance, it finished approximately 2.1 times faster 336 than ArcGIS and 1.7 times faster than TauDEM for the Savannah River basin, and approximately 2.5 times 337 faster than ArcGIS and 2.7 times faster than TauDEM for the Cooper River basin. We believe that this 338 speedup is because our approach is able to leverage pre-processed catchments rather than directly processing 339 DEMs. This supports our argument that using pre-processed national datasets, such as the NHD+, has 340 some distinct advantages over DEM processing approaches. However, as the number of catchments increases 341 (i.e. dataset size increases), the amount of time dedicated to geometry extraction from the database and 342 geometry merging, becomes more pronounced. This insight suggests that future work should focus on the 343 internal algorithms used in our approach. These basic benchmarks are an average of several simulation 344 runs in which raster pre-processing routines have been omitted. Therefore, we can expect the end-to-end 345

execution times to increase with inclusion of raster data preparation, whereas this will not effect the our approach.

348 4.2. Subwatershed Delineation

Another important feature of GIS software for hydrology applications is the ability to delineate subwatershed areas. Relatively little work is required to extend our approach to subwatershed delineation, which further demonstrates its flexibility. The same general methodology is applied to determine watershed areas, therefore only a small extension required to provide the additional functionality of deriving subwatershed areas from NHD+ catchments.

The extension is largely made to perform the procedure outlined in Section 3 over a list of outlet points 354 rather than a single location. This is illustrated by the loop mechanism in Figure 9, in which the geometry 355 and graph networks are queried for each outlet provided by the user. The upper and lower watershed areas 356 are determined for each outlet by following the methodology presented in Section 3. However, once all the 357 upper catchments have been identified for all outlets, they are isolated such that each catchment geometry 358 can only belong to one subwatershed, which in turn, is associated with one outlet. This is done to eliminate 359 redundant merging of catchment geometries. This can also be done in a less efficient manner by first merging 360 the upper and lower geometries for each outlet and then subtracting them from one and other to derive the 361 subwatersheds. The subwatershed boundaries are used to create a polygon output file, and the outlet points 362 are used to create a point output file. Together, these new files summarize the subwatershed topographic 363 characteristics of a particular region. 364



Figure 9: The procedure for subwatershed delineation using the hierarchical technique.

For comparison purposes, a widely used software suite is used to calculate the subwatersheds over the same area: Arc Hydro Tools. While Arc Hydro Tools is capable of performing a myriad of advanced

hydrology-related processing, we only leverage its subwatershed delineation functionality, outlined in Figure 367 10. First, a point shapefile is created using a list NHD+ output locations. Five additional attribute fields 368 are created to match the format of the Arc Hydro Tools "Batch Point" file that is required as input to the 369 Batch Subwatershed Delineation tool. Next, values for these attribute fields must be assigned, specifically, 370 BatchDone is set to 0 and SnapOn is set to 1. These values indicate that (1) batch processing has not 371 been completed and (2) that each outlet must be snapped onto the river network. Once complete, these 372 points are imported into the Arc Hydro Tools geodatabase using ArcCatalog. Next, the river network is 373 defined using the Stream Definition tool. This tool creates a raster product derived from the NHD+ flow 374 accumulation grid that consists of cells having an accumulation greater than a user defined value. Finally, 375 the Batch SubWatershed Delineation tool is executed to delineate basins for each of the outlets provided. 376



Figure 10: The procedure for subwatershed delineation within ArcGIS, using Arc Hydro Tools.

Both of these approaches were applied to delineate watersheds at 49 locations, corresponding to USGS 377 streamflow monitoring gages. Figure 11 shows the boundaries that were delineated using each of the afore-378 mentioned GIS approaches. The hierarchal technique finishes this operation in 69 seconds, whereas using 379 Arc Hydro this took 2 minutes. Again, discrepancies exist between the boundary calculated by Arc Hydro 380 Tools using raster computations and the boundary that was assembled from NHD+ catchments using the 381 hierarchical approach. Insets (i) and (ii) of Figure 11 illustrate the nature of the boundary differences we 382 found. In both cases, small catchment areas are left out of our calculation which contradicts the boundary 383 calculated directly from the terrain elevation using raster calculations. As described in Section 4.1, we 384 believe that this is a direct result of how the NHD+ dataset was created and the flow attributes therein. 385 In the first inconsistency, NHD+ derived boundaries don't exactly match the those calculated using raster 386 computations. Upon further inspection it appears that this has been corrected in version 2 of the NHD+ 387 dataset. The second case, however, is due to the river flow attributes that we used to trace upstream reaches 388 and subsequently catchments. This may be due to corrections that were made to the natural terrain to ac-389

³⁹⁰ count for the actual hydrography of the surface, or they could mistakes within the NHD+ dataset that must ³⁹¹ be corrected.



Figure 11: Subbasins were delineated at USGS streamflow stations using Arc Hydro Tools and the hierarchical approach. Boundary differences were found when compared with raster-based delineation and are illustrated by the lighter catchments in insets (i) and (ii).

³⁹² 5. Summary and Discussion

A watershed delineation technique was presented that uses existing GIS vector and raster data to resolve watershed boundaries for a wide range of spatial scales. It leverages freely available input data and open-source software which makes it easily accessible to a wide range of hydrologic scientists. Traditional watershed delineation approaches perform raster computations directly on DEM's, which inadvertently results in redundant computations (Djokic and Ye, 2000). Our approach is advantageous when large pre-delineated watershed datasets are available. When this is not the case, traditional DEM processing may be the pre-

ferred option. While these datasets can be created manually (Djokic and Ye, 2000), the encouragement of 399 programs similar to the USGS NHD+ by international agencies, would enable our algorithm to be easily 400 adopted for a wide range of hydrologic science applications. This approach will not replace traditional raster 401 processing algorithms which, among numerous other applications, is instrumental to deriving raster data 402 products required for model simulation (e.g. Quinnet al., 1995). However, its versatility lends it useful as a 403 hydrologic data processing tool that can be used to spatially summarize data attributes on a catchment or 404 subcatchment level. It can also be used as a boundary for search, collection, and/or extraction of simulation 405 input data (e.g. observation and spatial data) in a web based environment. 406

By leveraging pre-processed watershed catchment vectors, our technique offers an efficient solution to 407 watershed delineation. Furthermore, the input data used to construct watershed boundaries is pre-processed 408 by the USGS (i.e. NHD+), thus eliminating time intensive processing routines which have been necessary in 409 past works (e.g. Djokic and Ye, 2000; Arge et al., 2006; Danner et al., 2007). In addition, the NHD+ dataset 410 has been checked for accuracy by an interdisciplinary team of USGS and U.S. EPA scientists (Bondelid 411 et al., 2010). Moreover, the quality of watershed delineations should continue to improve with each release 412 of the NHD+ dataset. For example implementing our method on the newest version of the NHD+ (version 413 2) will automatically correct any errors that were detected in the previous version of the dataset. This is 414 significant because it streamlines the process of upgrading surface data by eliminating the need to download 415 numerous individual elevation raster grids which not only saves time, but also storage space. 416

Our approach for watershed delineation uses NHD+ data products to rapidly assemble watershed bound-417 aries. First, a small portion of the watershed is determined by tracing the grid cells upstream of the outlet 418 location. This upstream trace identifies all cells that contribute flow to the outlet, but that are limited to 419 the NHD+ catchment in which the outlet resides. This was made possible by borrowing from mathematical 420 graph theory and leveraging the NetworkX graphing library (Hagberg et al., 2008). The upper portion of the 421 watershed is determined using the flow relationships between the NHD+ digitized river reaches to identify all 422 upstream rivers and their respective catchments. The geometries for these catchments are then merged and 423 later combined with the lower portion of the watershed to complete the delineation. Because this technique 424 does not require grid processing, it can rapidly resolve a watershed boundary with a little computational 425 overhead. Furthermore, it was shown that our algorithm is advantageous for delineating watersheds at a 426 wide range of scales as well as delineating subwatersheds. 427

The application of our delineation approach demonstrates a method for delineating watersheds and subwatersheds at various scales in a time efficient manner. However, the results of Section 4 show that boundary differences exist between the watersheds delineated by our approach and those derived directly from rasters (i.e. ArcGIS, TauDEM, Arc Hydro Tools). The variations in watershed boundaries are a result of the datasets used to derive the NHD+ catchment boundaries and flow relationships, i.e. modified DEM and the WBD rasters used in combination with a watershed delineation algorithm designed to produce the

best agreement of available data (Johnston et al., 2009). This method allows experts to modify remotely 434 sensed terrain data to ensure that it is consistent with known field measurements, unlike traditional raster-435 based watershed delineation. However, the NHD+ dataset may contain processing errors such as those 436 outlined in Figure 11. In situations such as this, a manual correction may have been made to the dataset 437 to account for the actual hydrography of the surface (e.g. an obstruction to river flow), however it could 438 also be a mistake. We must consider these watershed boundary differences inherent to the dataset used by 439 our hierarchical algorithm, which in this case is the NHD+. If desired, a custom or alternate watershed 440 boundary dataset can be used in place of the NHD+ for greater quality assurance. For example, this work 441 uses the NHD+ version 1.0 dataset, however, a newer version of the dataset is now available that consists 442 the most accurate and up-to-date data. In fact, it appears that some of these boundary differences have of 443 been corrected in the NHD+ version 2. A significant advantage of our approach is its ability to easily adapt 444 to newer, more accurate, datasets without having to process large datasets. 445

Overall, our technique consists of a light-weight software algorithm that is implemented in the Python 446 programming language and mathematical graph theory to process watershed boundaries. NHD+ network 447 relationships are stored in serialized graphs, while spatial data are stored in a PostgreSQL spatial database 448 that leverages PostGIS functionality. However, this algorithm has also been adapted to leverage SQLite 449 database storage. This versatility demonstrates the portability of this technique, and as a result, it may be 450 a good candidate for remote hosting via web services or deployment in cloud environments. Future work will 451 investigate how this technique can be deployed as a web service to provide on-demand watershed delineations 452 by leveraging emerging cloud computing environments such as Microsoft Azure or Amazon Elastic Compute 453 Cloud (EC2). A service such as this could then be used to retrieve, process, and summarize input data for 454 models. In addition, this technique can be expanded upon to supply users with other NHD+ data products 455 or attributes consisting of new or summarized data. This is particularly useful when preprocessing data to 456 create model input files. 457

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461 7. Software Availability

The delineation software presented in this paper is available for download under the GNU General Public
 Licence V3 at https://bitbucket.org/Castronova/hierarchical-watershed-delineation.

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